1	Sensitivity of climate change detection and attribution to the
2	characterization of internal climate variability
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ABSTRACT

²³ 1. Introduction

At the centre of the climate change debate is the question of whether global warming can 24 be detected, and if that is the case, whether or not it can be attributed to anthropogenic 25 causes. Optimal ngerprinting is a powerful method of detection and attribution of climate 26 change (Hasselmann 1979, 1993; Hegerl et al. 1996) used widely in this area of research. In 27 essence, optimal ngerprinting is a multi-regression analysis that searches for the observed 28 temperature record response to external drivers or forcings such as changing levels of green-29 house gases, and aerosol loading (human-induced), volcanic activity and variations in solar 30 radiation (naturally induced). A key input in the procedure of this multiple regres-31 sion model is an estimate of the internal variability of the climate system, against which the 32 statistical signi cance of anthropogenic and natural signals must be compared. Hence, an 33 accurate depiction of this variability is crucial for the robustness of the results. 34

In this work we refer to internal variability as the characterization of the variations in the
 climate system that would occur in the absence of natural or anthropogenic forcings, solely
 due to the coupling of atmosphere, ocean, biosphere and cryosphere dynamics. In most cases
 Global Climate Models (GCMs) are used to estimate climate internal variability bilitldbili0d0 g 0tion of w

authors are careful to attempt the inclusion of model uncertainty in the regression model, 50

and test the robustness of their results under changes in the amplitude of the estimated internal variability, it is not clear whether or not other aspects of the internal variability

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where T is the global mean temperature and F is the external forcing. In Allen et al. (2009), the heat capacity $c = 7:22 \frac{W y}{m^2 - K}$ corresponds to the heat capacity of an ocean mixed layer of depth $d_{ml} = 75m$ assuming that the ocean covers 70% of the Earth surface. Best estimates for the climate feedback parameter and emective fnhJ/F1ctive75 data set to test the sensitivity of the results to the addition of the last seven years of obser vations up to 2012 (see section 3). Uncertainties in observed temperatures and estimates of
 forcings are ignored in this paper.

We additionally use the World Climate Research Programme (WCRP) CMIP3 multimodel archive of control simulations to study the internal variability simulated by the state of the art climate models (Solomon 2007). For completeness, we have used all the control simulations, regardless of drifts. We will comment on the e ect of drifts in the control segments on the nal results in Section 3.

(i) Detection and Attribution

The detection of climate change is the process of demonstrating that climate has changed in some well de ned statistical sense, without providing a reason for that change. Attribution of causes of climate change is the process of establishing the most likely causes for the detected change with some de ned level of con dence (Solomon 2007). In this work we aim to detect and attribute climate change by estimating the contribution to the observational record T_{obs} of each of the response temperatures T_i calculated using Eq.(1). In other words, we want to obtain the amplitudes $_{i}$ in the following expression:

$$T_{obs} = T + u; (2)$$

where T is a matrix with n + 1 columns including the n forced responses T_i, and a constant term to remove the mean. u is an stochastic term that represents the internal climate variability with covariance matrix is given by $= E(uu^y)$. Under the assumption that u is multivariate normal (Allen and Tett 1999), the optimal scaling factors, = (1; 2; :: n+1)are given by (Kmenta 1971):

$$^{h} = T^{y} {}^{1}T {}^{1}T^{y} {}^{1}T_{obs'}$$
(3)

147 and their variance :

$$V(^{\wedge}) = T^{y} \quad {}^{1}T \quad {}^{1}; \tag{4}$$

where y is used to denote the transpose of a matrix.

In this work, following standard detection and attribution studies, we consider the following external forcings: greenhouse gases, sulphates, volcanic and solar. It has long been recognized however, that the detection and attribution results are sensitive to the omission of potentially important forcings and/or internal modes of variability. Likewise, if signals that terize the global mean internal variability u explicitly as a stationary stochastic process. In
 other words, we formulate the detection and attribution problem as in Eq.(2) but with u a
 function of stochastic parameters that are estimated simultaneously with the scaling factors
 [^] using a minimum squared error algorithm.

The rst challenge is to choose an adequate stochastic representation for the internal vari-177 ability. The di culties nding the appropriate stochastic model are due to the uncertainties 178 in characterizing internal variability from the observational record, which as discussed be-179 fore, is contaminated by the external forcings and too short relative to the long time scales 180 potentially relevant to the current climate variability. In particular, in the observed record 181 it is not clear how to separate the decadal from centennial or even longer time scales (Percival 182 et al. 2001). Given these uncertainties in the characterization of the internal climate variabil-183 ity we choose to describe it using two models that span a wide range of plausible temporal 184 autocorrelations (Vyushin and Kushner 2009). This choice is important to address the fact 185 185

¹⁹⁹ between oceanic and atmospheric dynamics. In this framework, the faster dynamics of the ²⁰⁰ atmosphere can be modeled as white noise acting on the slower and damped dynamics of ²⁰¹ the ocean. Thus, the AR(1) is the simplest model that can explain the \weather " and the ²⁰² \climate" uctuations as two components of the internal variability. Mathematically, the ²⁰³ AR(1) is a stationary stochastic process that can be written as:

$$u_t = a_1 u_{t-1} + a_{0-t}$$
 (5)

where $E(u_t) = 0$, a_1 and a_0 are parameters, and t represents white noise, .e. $E(t_t) = t_t 0$. The autocovariance function of this process is determined by a_0 and a_1 as follows:

$$!_{AR1}() = \frac{a_0^2}{1 a_1^2} a_1^{j j}$$
(6)

where is the time lag. Notice that a_1 controls the decaying rate of the autocorrelation function and in that sense we can associate it to the **memory** of the system. On the other hand a_0 is related to the amplitude of the white noise in the system. From Eq.(6) the covariance matrix results:

$$_{i;j}^{AR} = \frac{a_0^2}{1 a_1^2} a_1^{ji} a_1^{jj}$$
(7)

Eq.(5) models the memory of the process such that at a given time t the state of the system is a linear function of the previous state (t 1) and some random noise with amplitude a_0^2 jittering, and hence moving the system away from equilibrium. The autocovariance of the process, Eq.(6), decays exponentially with time, so the system has always a much better memory of the near past than of the distant past. a_1 can take any value in the interval [0; 1), $a_1 = 0$ represents the limit in which the system is purely white noise, and a_1 / 1 is the the parameters a_1 and a_0 of the climate noise in Eq.(5) following the Hildreth-Lu method (Kmenta 1971).

223 (iii) Long memory process: FD

There is empirical evidence that the spectrum of global mean temperature is more complex than the spectrum of an AR(1) process (e.g. Huybers and Curry (2006)). Di erent power-law behaviors have been identi ed in globally and hemispherically averaged surface

$$u_t = \begin{pmatrix} 1 & B \end{pmatrix} \quad t: \tag{8}$$

²⁴⁷ where B is the backshift operator, i.e. $Bu_t = u_{t-1}$ (Beran 1994). The model is fully specied ²⁴⁸ by the parameters and the standard deviation _e of the white noise _t. The autocovariance ²⁴⁹ function is given by the equation:

$$!_{FD}() = \frac{\frac{2}{e}\sin()(1 - 2)(+)}{(+ 1 - 2)}$$
(9)

²⁵⁰ As a result the covariance matrix becomes,

$${}^{\text{FD}}_{i;j} = \frac{\frac{2}{e}\sin()(1 + 2)(ji + jj +)}{(ji + jj + 1)} :$$
 (10)

For large the autocorrelation function satis es $\lim_{1} !_{FD}() = j f^{2-1}$ (Beran 1994). From this expression one can see that the autocorrelation decays algebraically, thus the name "long memory". Since controls the decaying rate of the autocorrelation function it can be associated to the **memory** of the system, while _e is characterizes the amplitude of the white noise.

Similarly to the AR(1) case, we use this covariance matrix, Eq.(10), and Eq.(2) and Eq.(3) to simultaneously determine the scaling factors $_{i}$ and the parameters and $_{e}$ following the Hildreth-Lu method (Kmenta 1971).

259 3. Results

260 a. Robustness of detection statistics

In order to test the robustness of the detection statistics, we ind simultaneously the scaling factors _i and the stochastic parameters of the internal variability u, using generalized linear regression to solve Eq.(2). Notice that when u is modeled as an AR(1) or an FD process, the noise covariance matrix in Eq.(3) and Eq.(4) is given by Eq.(7) or Eq.(10) respectively. The best estimates of the scaling and noise parameters are chosen as those that minimize the residual white noise in u (Kmenta 1971). Using the Akaike Information Criteria
we nd that both models for u are equally skilful at representing the internal variability given
the observational record used in our analysis.

Fig.(1) shows the values of the optimal scaling factors with their 95% condence intervals using the AR(1) (grey line) and the FD (black line) models, when T_{obs} is the HadCRUT3 global mean temperature record for the period 1850-2005. In the detection and attribution approach, a signal is detected when the corresponding scaling factor is different from 0 with 95% condence, while the attribution of a signal requires condence intervals that include

given value of the z_{score} or, equivalently, the size of the condence interval, we aim to $rac{1}{1}$ nd 293 what is the proportion of cases where the scaling factor is di erent from 0. In particular 294 the value of the z_{score} that gives di erent from 0 in at most 5% of the cases determines 295 the 95% con dence interval. We nd that for the GHG signal the z_score is 2:22 in the case 296 of the AR(1) model and 2:45 in the case of the FD model. In addition, and since we expect 297 that due to the stochastic nature of the noise models there will be some uncertainty in the 298 determination of their parameters, the values of the noise model parameters estimated with 299 this Monte Carlo approach provide an estimate of the uncertainty of the best t noise model 300 parameters when regressing the forced responses on T_{obs} in Eq.(2). 301

Fig.(1) shows that for our detection model, the greenhouse gas signal is detected and attributed, the volcanic signal is only detected, and the solar signal is not detected nor attributed for both models of internal variability. In the case of the sulphates forcings, the result depends on the representation of the internal variability.

The robustness of the GHG signal detection can be analyzed using Fig.(2) when the 306 internal variability is characterized by the AR(1) model or by the FD model in the upper or 307 lower panels respectively. The horizontal and vertical axes show the white noise amplitude 308 and memory parameters respectively, and the contour lines indicate the signi cance level of 309 the scaling factor _{GHG}. The diamond symbol shows the best t of internal variability (for 310 each model) when the observed record T_{obs} is the HADCRUT3 data for the period 1850-311 2005. The uncertainty in the estimation of the best t, computed using the Monte Carlo 312 approach, is shown as the grey cloud of points. It is clear that even when taking into account 313 this uncertainty in the parameters, the signi cance of the detection of the greenhouse gas 314 signal is not a ected. 315

As expected, the signi cance of the greenhouse gas signal is lower when we represent the internal variability as an FD than as an AR(1) process. We nd that both stochastic models' best thave similar white noise amplitude, showing that statistically they are similarly good at explaining variability, given that this is the residual of the linear t. The bigger di erence

- ₃₂₀ between the two models arises in the memory parameter.
- In the case of the AR(1), a_1 is bounded between $a_1 = 0.25$ and $a_1 = 0.70$, and the best estimate is $a_1 = 0$:

the time it takes for the autocorrelation function to reduce to 1=e of its initial value (in analogy with the e-folding time for the AR(1) model). For the best t value of = 0:43 for instance, this calculation gives a much longer time than the length of the observational record (156 years). This suggests that, according to this model, in the 156 years long record all points are highly correlated. Overall, we nd that, despite the very di erent time scales that are relevant for the AR(1) and FD characterizations of internal variability, the GHG signal detection statistics is robust for both models.

One interesting question that can be explored using our results is how wrong one would have to get the model parameters of the internal variability in order to change the detection statement of the greenhouse gas signal. In the case of the AR(1) model we nd that the greenhouse gas signal would become not statistically signi cant in a world in which higher values of a_1 and/or a_0 were needed to describe internal variability. In the upper panel of Fig.2 we see that, to loose statistical signi cance, one would have to increase the time correlation characterized by a_1 to more than 0:8, or triple the white noise parameter a_0 .

Hence, the detection statistics for the AR(1) model is very sensitive to the memory parameter and relatively less sensitive to the amount of white noise in the process. Thus, in terms of the global mean temperature internal variability as simulated by GCMs, our ndings suggest that the relevant aspect that should be taken into account in a robustness test should be the models' ability to capture correctly the temporal correlations more than the total variance, which is in turn conditioned by their ability to capture the most relevant dynamical processes, their couplings and feedback mechanisms.

For the FD process we nd a di erent result. In the lower panel of Fig.(2) we can see that for the estimated _e there is no for which the process has a greenhouse gas scaling factor which is not statistically signi cant. Thus, this suggests that the greenhouse gases detection results are robust under changes in the memory parameter. In fact, for very high values of

401 b. CMIP-3 control runs

In this section we use the same techniques as above to evaluate the control simulations used in the detection and attribution of climate change included in the 4th Assessment Report of the IPCC. Our goal is to get some insight about the controls' potential limitations to estimate internal variability and how this might impact in the robustness of the detection and attribution statistics.

We take annual global mean temperature segments from the CMIP3 control simulations that have the same length as the observational record, 156 years, and t them to an AR(1)

equivalent to nding similar covariance matrices; hence this gure is consistent with our
previous ndings about the similarity in magnitude of the autocorrelation functions of the
tted internal variability to the 156 years observed record. It is clear that a much longer
time series is required to appreciate more signi cant di erences in the variability simulated
by the two stochastic models.

We can also analyze the link between the ability of a GCM to model di erent modes of 460 internal variability and the implications for the signi cance of detection and attribution. It 461 is clear from Fig.(4) that some control segments display peaks corresponding to the ENSO 462 signal with unrealistic high amplitudes, as shown by the high power at the 2-5 years fre-463 quency range. However, Fig. (2) shows that most of these control segments fall in the area of 464 the plots that correspond to a signi cant greenhouse gas signal. Consistently wit the ndings 465 in Allen and Tett (1999), this analysis suggests that an accurate depiction of all modes of 466 internal variability might not be required to ensure the robustness of the detection statistics 467 under our detection model. 468

Finally, our analysis point towards the need to develop a wider range of techniques to 469 assess the robustness of detection and attribution of climate change. The \consistency test" 470 described in Allen and Tett (1999) is equivalent to look at the power spectra of GCMs 471 runs and compare their (typically) decadal internal variability with the decadal internal 472 variability retained in the residuals of the t to the observed record. The aim of this test 473 is mainly to discard the possibility of over-attributing climate change to the anthropogenic 474 signal only because climate models under-represent decadal variability. However, studying 475 just the amplitude (or power) of internal variability in Fig.(4) does not give us information 476 about all the possible impacts that a model imperfection might have on the detection and 477 attribution statistics. Thus, there is a need to develop techniques that provide a way to 478 evaluate the impact of speci c modes of variability and their interactions, and not just their 479 amplitude, on the detection and attribution of climate change. Many interesting studies 480 have been developed recently (eg. DelSole et al. (2011)) but more work is needed. One 481

models are chosen to span a wide range of plausible temporal autocorrelation structures, and include the short-memory rst-order autoregressive (AR(1)) process and the long-memory fractionally di erencing (FD) process). We nd that, independently of the representation chosen, the greenhouse gas signal remains statistically signi cant under the detection model

APPENDIX A

⁵⁴² We use a HadCM3 control simulation of 1000 years to assess how the uncertainty of the ⁵⁴³ stochastic parameters depends on the length of the segment, and we refer to this as a nite

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List of Tables

⁶⁵⁹ 1 Scaling factors obtained from the linear regression when using HADCRUT4

	AR(1)	AR(1)	FD	FD
	1850-2005	1850-2012	1850-2005	1850-2012
VOL	0.46	0.48	0.51	0.53
SOL	2.26	2.03	1.14	0.99
GHG	0.94	0.71	0.91	0.66
SUL	2.47	1.44	2.04	0.93
VOL	0.54	0.52	0.55	0.53
SOL	0.98	1.24	0.58	0.83
ANT	0.76	0.71	0.81	0.73

Table 1. Scaling factors obtained from the linear regression when using HADCRUT4 observations for two time periods (1850 to 2005 and to 2012), and the forced temperature responses to VOL,SOL,GHG and SUL forcings , or to VOL, SOL and ANT forcings.

CCMA-CGCM3 CCCMA-CGCM3-1-T63 CNRM-CM3 CSIRO-MK3-0 GFDL-CM2-0 GFDL-CM2-1 **GISS-AOM GISS-AOM** GISS-Model-E-H GISS-Model-E-R IAP-FGOALS1-0-G IAP-FGOALS1-0-G IAP-FGOALS1-0-G INMCM3-0 **IPSL-CM4** MIROC3-2-HiRes MIUB-ECHO-G **MPI-ECHAM5** MRI-CGCM2-3 NCAR-CCSM3 NCAR-PCM1 UKMO-HadCM3

Table 2. CMIP-3 General circulation models used partly on the 4th IPCC Assessment report. The order on the table is the same as the numbering in previous gures.

690	4	Spectra from the individual GCM control simulations (gray), and the spectra	
691		of the residuals of the linear $$ t to $T_{obs}\!\!:T_{obs}^{}$ ^T , when the internal variability	
692		is modeled as an AR(1) (thick grey line) and an FD (black line) process. We	
693		use a logarithmic scale in the horizontal axis (period) and the vertical axis	
694		(spectral density).	37
695	5	AR(1) results of estimating a_1 (upper panel) and a_0^2 (lower panel) as a function	
696		of the length of the control segment sampled from the 1000 years long HadCM3	
697		control run.	38
698	6	FD results of estimating (upper panel) and $_{e}$ (lower panel) as a function of	
699		the length of the control segment sampled from the 1000 years long HadCM3	
700		control run.	39
701	7	Upper panel: correlation between the memory parameter of both stochastic	
702		models, values (vertical axis) versus a_1 values (horizontal axis) obtained	
703		from the CMIP3 control segments considered in our analysis. Lower panel:	
704		same for the white noise parameter of both stochastic models, $_{\rm e}$ (vertical	
705		axis) versus a_0^2 (horizontal axis). Each color corresponds to a di erent GCM.	40

Fig. 1. The 95% con dence intervals of the scaling factors _i derived from the multiregression of observed temperature changes onto the BDM estimates of the forced responses. The internal variability is represented by an AR(1) model (grey line) or an FD model (black line) for the period 1850 2005

Fig.



Fig. 3. The 95% con dence intervals of the scaling factors _i derived from the multiregression of observed temperature changes onto the BDM estimates of the forced responses to the three signals VOL, SOL and ANT (top panels) and VOL, SOL, GHG and SUL (bottom panels). The internal variability is represented by an AR(1) model (grey line) or an FD model (black line) for the period 1850 2005 (left hand side) and the period 1850 2012 (right hand side), using HadCRUT4



Fig. 4. Spectra from the individual GCM control simulations (gray), and the spectra of the residuals of the linear t to T_{obs} : T_{obs} T , when the internal variability is modeled as an AR(1) (thick grey line) and an FD (black line) process. We use a logarithmic scale in the horizontal axis (period) and the vertical axis (spectral density).



Fig. 5. AR(1) results of estimating a_1 (upper panel) and a_0^2 (lower panel) as a function of the length of the control segment sampled from the 1000 years long HadCM3 control run.



Fig. 6. FD results of estimating (upper panel) and $_{\rm e}$ (lower panel) as a function of the length of the control segment sampled from the 1000 years long HadCM3 control run.



Fig. 7. Upper panel: correlation between the memory parameter of both stochastic models, values (vertical axis) versus a_1 values (horizontal axis) obtained from the CMIP3 control segments considered in our analysis. Lower panel: same for the white noise parameter of both stochastic models, $_{e}$ (vertical axis) versus a_0^2 (horizontal axis). Each color corresponds to a di erent GCM.