Using shadowing ratios to evaluate data assimilation techniques

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Successful prediction of complex process has a huge value to society, particularly in areas like meteorology where it can a ect many lives. A modeller of chaotic dynamical systems however, faces big challenges. Even with a perfect model of the dynamics of a system, if past observations are clouded by measurement error, accurate predictions at long lead times will be hard to achieve. Data assimilation is the process of improving on these ”noisy” observations by attempting to recover the true trajectory. Shadowing lengths measure how long the model trajectory stays close to reality, for a good model prediction of the future, clearly we desire that these lengths should be as long as possible. We show how we can use ratios of these values to compare models and in particular, assimilation techniques.

2 Shadowing

A shadowing length is time a trajectory stays consistent with the observational noise of future observations. For forecasting purposes it is desirable for these lengths of time to be as long as possible.

definition 1 A model trajectory shadows for time if it is consistent with the observational uncertainty at all times t, $0 \leq t \leq$ **.**

The conditions of a trajectory shadowing observations depend on the observational noise model which we assume to be known. For bounded noise, these condtions are straightforward, for example if the noise model is distributed as $U(- \ , \ )$, the noise is bounded by . Therefore, a trajectory stops shadowing the observations at the smallest value of i such that $|x_i - S_i| >$ where x_i and S_i are points from our