Thorpex 2004

Scoring Probabilistic Forecasts

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- Why are probabilistic forecasts important?
- What is a skill score?
- What is a ***proper*** skill score?
- The importance of being proper
- Examples

Problem: We want to forecast an observable $T_{\mathbf{n}}$ (e.g. temperature), where n is the time.

- We issue probabilistic forecasts: $p_{\mathbf{n}}(T)$ = {Pobability of $T_{\mathbf{n}}$ = T}

- Usually $p_{\mathbf{n}}$ is built upon some related side information (past observations, weather model simulations)

- Does ***not*** mean $p_{\mathbf{n}}(t)$ the probability of $T_{\mathbf{n}}$ ***given*** that side information

Motivation

- End users don't want to ***know*** $T_{\mathbf{n}}$, they want to base ***decisions*** on $T_{\mathbf{n}}$ (and none of them care about models' 500mB height)

- To take reasonable action, the ***risk*** of taking that action must be factored into that decision

- To do that, information about the ***uncertainty*** of $T_{\mathbf{n}}$ must be known

We need a ***general skill score***, that takes into account the probabilistic character of the forecasts and that is relevant to many different users (incl. model developers, meteorologists).

- A skill score is a function $\mathcal{S}(p\ t)$

- The empirical skill is a sample mean:

$$S = \frac{1}{N}\sum_{n} \mathcal{S}(\underbrace{p_n}_{\text{Our forecast}}\ \underbrace{T_n}_{\text{Reality}})$$

What should skill scores actually measure?

Probabilistic forecasts

A good probabilistic forecast should have:

- Reliability – Looking at those days where a probability $p_n = r$ of rain is forecasted, a fraction r of them should have rain

- Sharpness – High probability is issued to events that acctually happen to occur

Skill scores should take this into account, since we ***believe*** that a probabilistic forecast having these properties is good for a ***multitude*** of speci c problems.

op co

Propriety is the key property for a skill score

- Assume

op co

Propriety is a property of the Skill Score alone, what the actual truth is doesn't matter

op co and a pn p y

Two statements for proper scores:

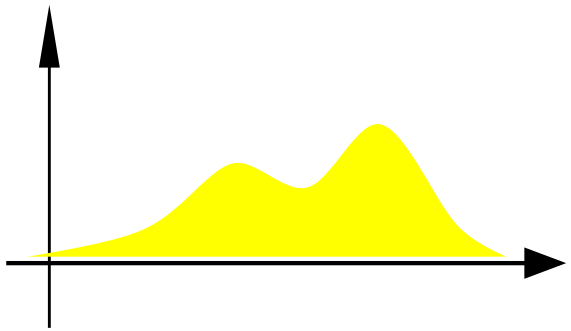
If, given the available information, a reliable forecasts exists, it would yield a maximum score

Of two equally reliable forecasts, the sharper one would score higher

An xa p a o

Bet on temperature at London Heathrow. Objective: Maximize the expected return rate

- Strategy: Distribute your wealth



An xa p a o

- Actual wealth grow rate: S = $\frac{1}{N}\sum_n \log p(T_n)$ + something that depends on the odds only

Can this be used as a skill score?

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- The ***Ignorance Skill Score*** is

$$I(p) = -\log(p_{\mathbf{n}}(T_{\mathbf{n}}))$$

- The Ignorance is proper, local and smooth

- The Ignorance is the ***only*** proper, local and smooth score for continuous forecasts (Good 1952, Gneiting & Raftery 2004)

- The ***Linear Score*** or p ***Score*** $\mathcal{S}(p_{\mathbf{n}}\ t) = p_{\mathbf{n}}(t)$ is ***improper***

- The RMS error depends only on some moments and therefore is ***not*** strictly proper

- Many proper ***nonlocal*** Scores have been suggested and used (see talk by Zoltan Toth about CRPS)

Conclusion

- End users need probabilistic forecasts to make better decisions
- We need skill scores that measure desirable properties of probabilistic forecasts
- We need to use proper scores, since improper scores give misleading answers – we would reject even the optimal forecast
- There are only a handful of essentially different proper skill scores (see www.dime.lse.ac.uk)