Matching Workers

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June 13, 2016

Abstract

This paper studies the matching of workers within the …rm when the productivity of workers depends on how well they match with their co-workers. The …rm acts as a coordinating device and derives value from this role. It is shown that a worker's contribution to …rm value changes over time in a non-trivial way as co-workers are replaced by new workers.

The paper derives optimal hiring and replacement policies, including an optimal stopping rule, and characterizes the resulting equilibrium in terms of worker ‡ows, …rm output and the distribution of …rm values. Simulations of the model reveal a rich pattern of worker turnover dynamics and their connections to the resulting …rm values distribution.

The paper stresses the role of horizontal di¤erences in worker productivity, which are di¤erent from vertical, assortative matching issues.

Non-Technical Summary

How does the value of the …rm depend on the value of its workers? When one considers …rms that have little physical capital –such as IT …rms, software development …rms, investment banks and the like –the neoclassical model does not seem to provide a reasonable answer. The …rm has some value that is not manifest in physical capital. Rather, 'organization capital' may be a more relevant concept in this context. One aspect of the latter form of capital is the formation of teams and this is the issue taken up in the current paper. We ask how workers a¤ect each other in production and how this interaction a¤ects …rm value. The current paper thus o¤ers an exploration of "organizational rent." The paper studies the value of …rms and their hiring and …ring decisions in an environment where the productivity of the workers depends on how well they match with their co-workers and the …rm acts as a coordinating device. This role of the …rm is what generates value.

The paper derives optimal hiring and worker replacement policies and characterizes the resulting equilibrium in terms of employment and the distribution of …rm values. A key result is the derivation of an optimal worker replacement strategy, based on a productivity threshold that is de…ned relative to the other workers. The derivation is non-trivial and underlines the importance of worker complementarities in productivity. Thus the model is not equivalent to one with shocks to individual workers or to job-worker pairings.

This replacement strategy (interacted with other worker separation and with …rm exit) generates rich turnover dynamics. The resulting …rm values distribution are found to be – using illustrative simulations – non-normal, with negative skewness and negative excess kurtosis. This shape re‡ects the fact that, as …rms mature, there is a process of forming good teams on the one hand and the e¤ects of negative separation and exit shocks on the other hand.

1 Introduction

This replacement strategy, interacted with exogenous worker separation and …rm exit shocks, generates rich turnover dynamics. The resulting …rm values distribution are found to be – using illustrative simulations – non-normal, with negative skewness and negative excess kurtosis. This shape characteristics are random at the stage at which the …rm decides on whom to hire.

A common way to model worker heterogeneity, and which we use in this paper, is to attribute to each worker a location in a metric space, and apply a distance measure to capture the di¤erences between the workers. In order to ensure that workers with di¤erent locations to be equally attractive in expected terms, we have to put restrictions on the space in which workers are located. A common way to obtain this is to assume that a worker has a location on a Salop (1979) circle and that workers are allocated uniformly on the circle.[2] In this case, the distribution of the distance from a worker to a co-worker randomly placed on the circle is independent of the worker's location. Note that this is not the case if the workers are uniformly allocated on a line segment, in which case a worker at the middle of the segment on average has a shorter distance to a randomly allocated co-worker than a worker close to the end point. More generally, in an n dimensional Euclidean space, an $n \quad 1$ dimensional sphere will also have the property that the distribution of the distance to a randomly placed co-worker will be independent of a worker's location on the sphere. However, in this case the distribution of the distance to a randomly placed co-worker is no longer uniform. In the discussion section we argue that a higher-dimensional sphere may be a convenient location space if there are more than three workers.

In what follows we therefore attribute to all workers a position on a Salop circle, with their placement randomly and independently drawn from a uniform distribution. Any new worker placement will be drawn independently from the same distribution. Note that if two workers are close on the circle, a third worker will either be close to or far away from both of the workers. Hence the distances from the third, new worker, to each of the existing workers are workers are positively correlated. This seems reasonable. The productivity of a team of workers is assumed to depend negatively on the distance between the workers.

Let $\quad = \frac{1}{\quad}$

2.2 Workers' Productivity and Interactions

We now turn to a formal description. The three workers are located on the unit circle. The one in the middle (out of the three) is the j worker who satisfies

$$\min_j \sum_{i=1}^{3} d_{ij} \tag{1}$$

where d_{ij} is the distance between worker i and j, and $d_{ii} = 0$. We shall define two state variables $_1$; $_2$ as follows:

$$_1 = \min_{i;j} d_{ij} \tag{2}$$

$$_2 = \min_j d_{kj} \ ; k \neq i \ ; j \quad i \ ; j = \arg\min_{i;j} d_{ij} \tag{3}$$

The first state variable $_1$ expresses the distance between the two closest workers. The second state variable $_2$ expresses the distance between the third worker and the closest of the two others.

The following figure illustrates:

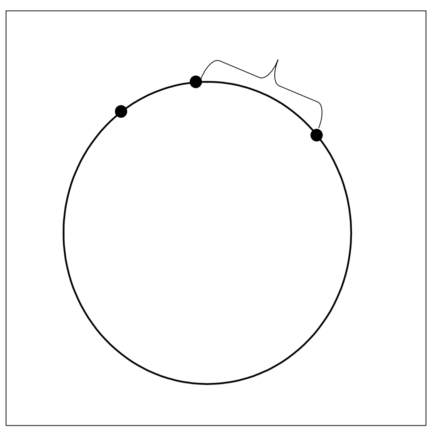

Figure 1: The State Variables

Every period, each worker works together with both co-workers to produce output. Output depends negatively on the distance between the workers. When measuring the distance between two peripheral workers, we assume that it is measured on the segment that goes through the middle man, not the other way around the circle (even if that is shorter). Partly this is meant to capture the structure of a team, that it needs a common ground.

Partly it is done for convenience, as it simplifies the algebraic expressions somewhat. It is not important for the results[3].

The firm's total output is written as a linear additive function:

$$Y = ye \quad 2(\ _1 + \ _2)$$

We assume that wages are independent of match quality. This is consistent with a competitive market where firms bid for *ex ante* identical workers prior to knowing the match quality. The profits ( ) of the firm are then given

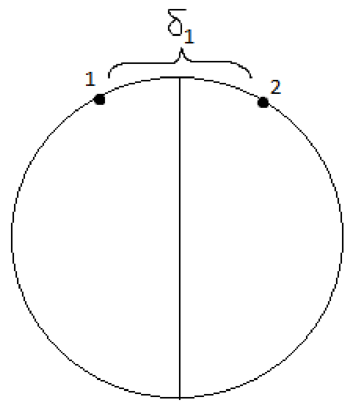

Figure 2: Incumbent Workers

From Figure 2 it follows that can be characterized as follows:

1. With probability 1 3 $_1$, $^0_1$ = $_1$ and $^0_2$ unif[$_1$; $\frac{1 \quad 1}{2}$]
2. With probability 2 $_1$, $^0_1$ unif[0; $_1$] and $^0_2$ = $_1$
3. With probability $_1$, $^0_1$ unif[0; $_1$=2] and $^0_2$ = $_1$ $^0_1$

Note that the transition probabilities, and hence continuation values when replacing, are a function of $_1$ and thus are independent of $_2$. Hence only $_2$ in‡uences continuation values in states where the …rm is not replacing. That is, as follows from the de…nition of pro…ts (equation 4), the continuation value of inaction is a function of ($_1$ + $_2$).

2.3 Microeconomic Stylized Facts

The afore-going set-up aims at capturing properties that have been found in empirical micro-studies of team production and complementarities within …rms. To note just a few examples: Hamilton, Nickerson and Owan (2003) …nd that teamwork bene…ts from collaborative skills involving communication, leadership, and ‡exibility to rotate through multiple jobs. Team production may expand production possibilities by utilizing collaborative skills. Turnover declined after the introduction of teams. Bresnahan, Brynjolfsson and Hitt (2002) study U.S. evidence and stress the importance of complementarities between workplace organization (and organizational changes) and computerization. Garicano and Wu (2012) discuss how performing complementary tasks leads to the formation of an homogenous team.

A recent study, undertaken by MIT's Human Dynamics Laboratory, collected data from electronic badges on individual communications behavior in teams from diverse industries. The study, reported in Pentland (2012), stresses the huge importance of communications between members for team productivity. In describing the results of how team members contribute to a team as a whole, the report actually uses a diagram of a circle (see Pentland (2012, page 64)), with the workers placed near each other contributing the most. The findings state that face to face interactions are the most valuable form of communications, much more than email and texting, thereby emphasizing the role of physical distance.

2.4 A Detour: One-Dimensional Optimal Stopping

Before we continue, we will briefly examine our model with only two workers. Our model then collapses to an optimal stopping model as in McCall (1970). It can also be viewed as a simplified version of the Jovanovic (1979 a,b) model, where the entrepreneur learns the worker type after one period[4].

The owner of a firm needs two workers to produce. Analogous with the

Inserting for V (") and manipulating gives that " solves[5]

$$\frac{**2}{r} \quad (\frac{1}{4} \quad **) \quad c = 0 \qquad (6)$$

The …rst term re‡ects the expected gain from replacing in terms of lower distances in all periods if the draw is good. The second term re‡ects the

Note that the existence of a stopping rule of this form is not obvious. For example, suppose we formulate the stopping rule in terms of total distance $X = 2(\ _1 + \ _2)$ rather than in terms of $\ _1$ and $\ _2$, that is, stop if $X \quad X$ for some $X > 0$. Such a stopping rule cannot be optimal. To see this, note that (i) for a given X, the pay-o¤ if stopping is independent of the decomposition of X into $\ _1$ and $\ _2$, and (ii) the pay-o¤ if replacing for a given X is decreasing in $\ _1$ (see below). Hence it cannot be optimal to apply a stopping rule under which stopping depends only on total distance.

By the logic of equation (5), note that in the stopping region, we have that

$$V(\ _1 + \ _2) = (y \quad 2(\ _1 + \ _2))\frac{1+r}{r} \tag{7}$$

Outside the stopping region, the continuation value depends only on $\ _1$. De…ne $\overline{V}(\ _1) \quad EV(\ ^0_1;\ ^0_2)j\ _1$ as the expected continuation value if the …rm chooses to replace. The value function in the case of replacement can then be written as:

$$V(\ _1;\ _2) = y \quad 2(\ _1 + \ _2) + \ \overline{V}(\ _1) \tag{8}$$

We start by showing an important property of the value function.

Lemma 1 $\overline{V}(\ _1 + \ ) > V(\ _1) \quad 2 \ \frac{1+r}{r}$

Proof. Consider replacement in two cases in which the distances between the remaining workers are $\ _1$ and $\ _1 + \ $, respectively. We refer to the two cases as the $\ _1$-case and the $\ _1 + \ $-case, respectively. The expected pay-o¤s only depend on the distances between the agents, and not on their exact location on the circle. Without loss of generality, we can therefore assume that in both cases, the two workers are located symmetrically around the north pole, and that the draw of the new worker is the same in the two cases. In what follows we assume that the …rm in the $\ _1 + \ $ case follows exactly the same replacement strategy as the …rm in the $\ _1$ case (replaces the worker on the left hemisphere whenever the optimal strategy in the $\ _1$ case prescribes so, the same for the worker on the right hemisphere, and stops searching after the same draws of location). We refer to it as the replication strategy. This is clearly in the choice set of the …rm. Hence if we can show that the replication strategy gives the …rm in the $\ _1 + \ $ case a pro…t that is strictly greater than $V(\ _1) \quad 2 \ \frac{1+r}{r}$, the proof is complete.

Let $\ ^n_1$ and $\ ^n_1$ denote the state variable in the two cases after n periods, and let $\ ^n \quad \ ^n_1 \quad \ ^n_1$. De…ne $\ ^n_2$ and $\ ^n_2$ correspondingly. Consider …rst the case with $n = 1$. Let $\ _{tot}$ be de…ned as $\ _{tot} \quad \ ^1_1 + \ ^1_2 \quad \ ^1_1 \quad \ ^1_2$.

It follows that the di¤erence in output the …rst period after replacement is equal to 2 $_{tot}$. There are three possibilities:

(i) The new worker is located below the workers in the $_1$ +

The lemma captures the essence of replacement: it makes a bad draw less costly than without replacement, since the …rm can always make a new draw. For any $\ _1$, $\ _2$, let $D(\ _1;\ _2)$ denote the value of replacing less the value of stopping, i.e., from equation (7) and (8),

$$D(\ _1;\ _2) \quad y \quad 2(\ _1 + \ _2) + \ V(\ _1) \quad (y \quad 2(\ _1 + \ _2))\frac{1 + r}{r}$$

$$=$$

The …nding that $_2(\ _1)$ is strictly decreasing in $_1$ deserves a comment. At $_1 = \ _1$, $_2(\ ) = \ _1$. As $_1$ decreases below $_1$, $_2(\ _1)$ increases above $_1$. This rules out the possibility of a non-monotonicity in stopping behaviour, in the sense that a good draw that reduces $_1$ makes the …rm more choosy and induces it to replace more. Appendix A shows the full derivation of $\ $:

As will become clear below, a …rm will replace for large values of $_1$ provided that r and c are not too big.

3.2 Characterizing the Stopping Rule

In this section we will characterize $\bar{\ }_2(\ _1)$. Now

$$\begin{aligned} V(\ _1;\ _2) &= \ (\ _1;\ _2) + \ \max[V(\ _1;\ _2); \overline{V}(\ _1) \ \ c] \\ &= y \ \ 2(\ _1 + \ _2) + \max[\frac{y \ \ 2(\ _1 + \ _2)}{r}; \frac{\overline{V}(\ _1) \ \ c}{1 + r}] \end{aligned} \tag{10}$$

It follows directly from proposition 4 in Stokey and Lucas (1989, p.522) that the value function exists. By de…nition the optimal stopping rule must satisfy

$$V(\ _1; \bar{\ }_2(\ _1)) = \overline{V}(\ _1) \ \ c$$

Or (from equation (10))

$$\frac{y \ \ 2(\ _1 + \bar{\ }_2(\ _1))}{r} = \frac{\overline{V}(\ _1) \ \ c}{1 + r} \tag{11}$$

Let E^{jx} denote the expectation conditional on x. Intuitively, the expected value of replacement, $\overline{V}(\ _1)$, is given by:

With probability $_1$ the new worker will fall between the two incumbents, and the total sum of distances between all workers will be $2\ _1$

Summing up, the total expected sum of distances between all workers after replacement is:

$$2\ E^{j}\ _1\ \ {}^0_1 + {}^0_2\ = 2\ \ {}^1$$

3. Finally we show that

$$\Pr(\ {}^{0}_{2} > \bar{\ }_{2}(\ _{1}))\frac{\overline{V}(\ _{1}) \quad c}{1 + r} = (1 \quad _{1} \quad 2\bar{\ }_{2})\frac{\overline{V}(\ _{1}) \quad c}{1 + r}$$

This comes from the fact that with probability $(1 \quad _1 \quad 2\bar{\ }_2)$ the new worker is above the $\bar{\ }_2$ threshold. The …rm will keep replacing and pay the cost c again.
We have thus fully derived equation (13).

Let us write:

$$\begin{aligned} & (\ _{1} + 2\bar{\ }_{2})y \quad 2\bar{\ }_{2}(2\ _{1} + \bar{\ }_{2}) \quad 2\ {}^{2}_{1} \\ = \ & (\ _{1} + 2\bar{\ }_{2})(y \quad 2(\ _{1} + \bar{\ }_{2})) + 2\bar{\ }^{2}_{2} + 2\ _{1}\bar{\ }_{2} \end{aligned}$$

Hence we can re-write (13) as follows:

$$\overline{V}(\ _{1}) \quad = \quad y \quad (\frac{1}{2} + \ _{1} + \frac{\ {}^{2}_{1}}{2}$$

which is the LHS of (15).

The RHS of (15) represents the gains from replacement associated with lower costs in all future periods if the draw is good.

With probability $_1$ the new worker will be between the two existing workers who have a distance of $_1$ between them. The total distance between the three workers is $2\,_1$: Existing total distance is $2(\,_1 + \bar{}_2)$, and the savings in distance is thus $2\bar{}_2$. Multiplying this with the probability of the event $;\,_1$, gives the …rst term in the nominator of the RHS of (15).

With probability $2\bar{}_2$ the worker is not between the existing workers but within a distance of $\bar{}_2$ from one of them. The expected distance of the new worker to the nearest existing worker is $\bar{}_2=2$ and to the other existing worker it is $_1 + \bar{}_2=2$. The

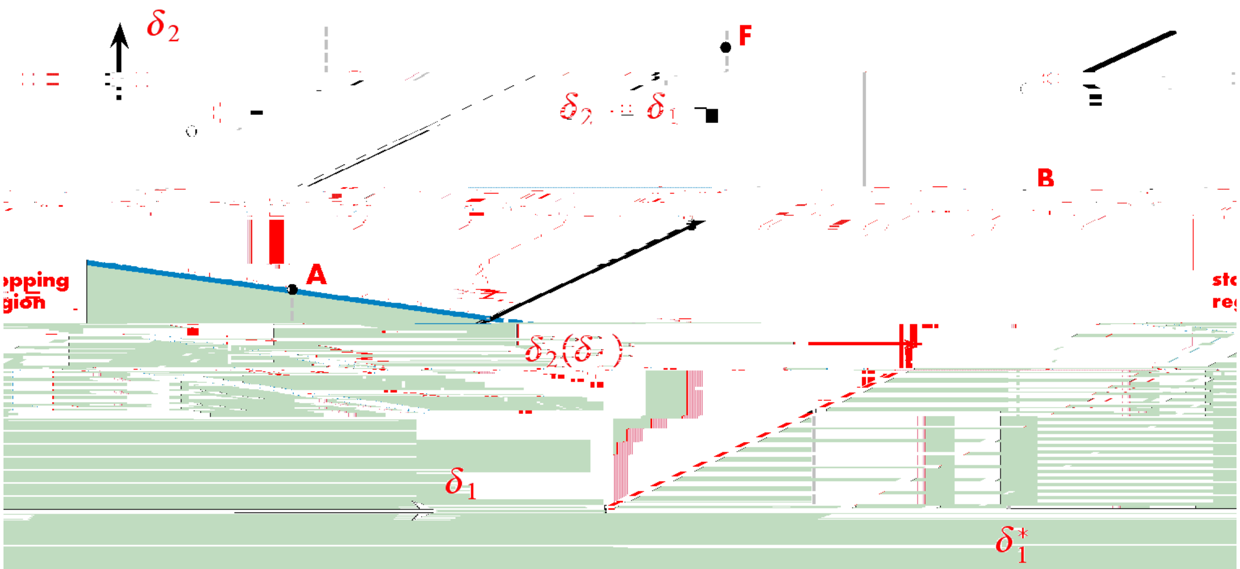

Figure 4: Optimal Policy

The space of the …gure is that of the two state variables, δ_1 and δ_2: The feasible region is above the 45 degree as $\delta_2 \geq \delta_1$ by de…nition. The downward sloping line shows the optimal replacement threshold $\bar{\delta}_2$ as a function of δ_1:

With the replacement of a worker, the …rm may move up and down a vertical line for any given value of δ_1 (such as movement between A, B and C or between D, E and F). If the replacement implies a lower value of δ_1, this vertical line moves to the left. This is what happens till the …rm gets into the absorbing state of no further replacement in the shaded triangle formed by the $\delta_1 = \bar{\delta}_2(\delta_1)$ point, the intersection of $\bar{\delta}_2(\delta_1)$ line with the vertical axis, and the origin ($\delta_1 = \delta_2 = 0$).

The following properties of turnover dynamics emerge from this …gure and analysis:

(i) At the NE part of the δ_1 δ_2 space, δ_1; δ_2 are relatively high, output is low, and the …rm value is low. Hence the …rm keeps replacing and there is high turnover. Note that some workers may stay for more than one period in the …rm when in this region. The dynamics are leftwards, with δ_1 declining, but δ_2 may move up and down.

(ii) Above the $\bar{\delta}_2(\delta_1)$ threshold, left of δ_1, newcomers may still be replaced, but veteran workers are kept.

(iii) In the stopping region there is concentration at a location which is random, with a flavor of New Economic Geography agglomeration models. Thus firms specialize in the sense of having similar workers. There is no turnover, and output and firm values are high.

(iv) Policy may affect the regions in $_1$ $_2$ space via its effect on c: The discount rate affects the regions as well.

(v) These replacement dynamics imply that the degree of complementarity between existing workers may change. This feature is unlike the contributions to the match of the agents in the assortative matching literature, where they are of fixed types.

3.4 Closing the Model

Our main purpose in this paper is to study replacement, and this can be done in partial equilibrium. Still, for completeness we demonstrate how the model can be closed by endogenizing the wage w and pin it down by a free entry condition. There are costs $K \quad 3c$ to open a firm. A zero profit condition pins down the wage ($w = \frac{W}{3}$):

$$E^{j \ 1 \ 2} V(\ _1; \ _2; w; y; c) = K \tag{16}$$

As we have seen, the hiring rule is independent of w (since it is independent of y). If y is sufficiently large relative to K, we know that $E^{j \ 1 \ 2} V(\ _1; \ _2; w; y; c) > K$, and there exists a wage w that satisfies (16). A formal proof of existence, as well as sufficient conditions on the parameters that ensure existence and production in each period, is given in Appendix C.

4 Exogenous Replacement

We now allow, with probability , for one worker to be thrown out of the relationship at the end of every period. If the worker is thrown out, the firm is forced to search in the next period.[6] Thus, if the replacement shock hits, one of the workers, chosen at random, has to be replaced. The firm can only hire one worker in any period, and hence will not voluntarily replace a second worker if hit by a replacement shock. If the shock does not hit, the firm may choose to replace one of its workers or not.

[6] With minor adjustments of the model, replacement can be interpreted as a change of position on the circle of one worker, due to learning to work better with other workers or, the opposite, the "souring" of relations.

Suppose one worker is replaced by the …rm as above. The transition probability for $(_1; _2)$ was denoted by $(_1)$, and depends only on $_1$. We refer to this as the basic transition probability.

The forced transition probabilities are the transition probabilities which occur when one worker is forced to leave, to be denoted by $^F(_1; _2)$. Which of the three incumbent workers leaves is random: with probability 1=3 the least well located worker leaves, in which case the transition probability is $(_1)$; with probability 1=3, the second best located worker leaves, in which case the transition probability is $(_2)$; with probability 1=3, the best located worker leaves, in which case the distance between the two remaining workers is $\min[_1 + _2; 1 \quad _1 \quad _2]$. It follows that the forced transition probabilities can be written as

$$^F(_1; _2) = \frac{1}{3} (_1) + \frac{1}{3} (_2) + \frac{1}{3} (\min[_1 + _2; 1 \quad _1 \quad _2]) \qquad (17)$$

With exogenous replacement, the probability distributions for $^0_1$ and $^0_2$ depend on both $_1$ and $_2$, not just $_1$ as above. The Bellman equation reads:

$$\begin{aligned} V(_1; _2) \quad = \quad & (_1; _2) + \quad [\ E \quad V_1(^0_1; ^0_2) \quad c] \\ & +(1 \quad) \quad \max[V(_1; _2); V(_1) \quad c] \end{aligned} \qquad (18)$$

The …rst term in the bracket shows the expected NPV of the …rm if the …rm is hit by a replacement shock. The second term in the bracket shows the expected NPV if the …rm is not hit by a replacement shock. It follows directly from Proposition 4 in Stokey and Lucas (1989, p. 522) that the value function exists. Furthermore, due to continuity, we know that the optimal replacement strategy can be characterized by an optimal stopping rule provided that is small.

5 The Model in the Context of the Literature

The paper bears (limited) similarity to Kremer's (1993) O-ring production function model. The similarity pertains to the importance attributed to the idea of workers working well together. In that model …rms employ workers of the same skill and pay them the same wage. In this set-up quantity cannot substitute for quality. But the models di¤er in their treatment of the matching of workers: in Kremer (1993) there is a multiplicative production function in workers/tasks and this underlies their complementarity. In the current paper there is explicit modelling of the match between workers, formalized as random state variables, which realization elicits the firm's optimal worker replacement policy.

The paper stresses the role of horizontal differences in worker productivity, as opposed to vertical, assortative matching issues. The literature on the latter –see the prominent contributions by Eeckhout and Kircher (2010, 2011), Shimer and Smith (2000), and Teulings and Gautier (2004)), and the overview by Chade, Eeckhout, and Smith (2016) –deals with the matching of workers of different types. Key importance is given to the vertical or hierarchical ranking of types. These models are defined by assumptions on the information available to agents about types, the transfer of utility among workers (or other mating agents), and the particular specification of complementarity in production (such as supermodularity of the joint production function). In the current paper, workers are ex-ante homogenous, there is no prior knowledge about their complementarity with other workers before joining the firm, and there are no direct transfers between them. In similar vein, the models of Garicano and Rossi-Hansberg (2006) and Caliendo and Rossi-Hansberg (2012), whereby agents organize production by matching with others in knowledge hierarchies, stresses the vertical dimension of worker communication. In terms of those models, the current paper is relevant for the modelling of team formation at a particular hierarchical level. Thus these approaches are complementary to ours.

The paper has points of contact with papers in the search literature. We exploit the idea of optimal stopping, as in McCall (1970) and the rich strand of search literature which followed (see McCall and McCall (2008), in particular chapters 3 and 4, for a comprehensive treatment). The existing literature does not cater, however, for the key issue examined here, namely that of worker complementarities. Conceptually this is an important distinction, and it allows us to analyze team formation in detail. Technically it also gives rise to new challenges. Total match quality (or output) depends on two variables that are stochastic ex ante, the distances from the best placed worker to each of her two co-workers. At the same time the firm replaces only one worker at a time. This creates a new dimension to the optimal stopping problem, which, in contrast to most earlier studies, now depends on a state variable (the distance between the two closest workers who are not replaced in a given round). Furthermore, optimal stopping behaviour depends on this state variable in a non-trivial way, and it is not even obvious from the outset that a simple optimal stopping rule exists.

Our paper shares some features with the search model of Jovanovic (1979 a,b): there is heterogeneity in match productivity and imperfect information ex-ante (before match creation) about it; these features lead to worker turnover, with good matches lasting longer.[7] But it has some important differences: the Jovanovic model stresses the structural dependence of the separation probability on job tenure and market experience. There is growth of firm-specific capital and of the worker's wage over the life cycle. In the current model the workers do not search themselves and firms do not offer differential rewards to their workers. But the Jovanovic model does not cater for the key issue here, namely that of worker complementarities.

worker locations. The main reason why we use the Salop circle is that it conveniently allow the distances from a given worker to a randomly placed co-worker to be independent of the worker's location. Hence, this modelling technique readily implies that the workers' location, *ex ante*, does not influence his expected contribution to a team. As already indicated in the text, this property does not carry over to a location on a line segment. A worker located close to the middle of the line will on average be closer to randomly allocated co-workers than a worker located close to the an end point. In addition, the Salop circle easily captures the notion that if A works well with B and B with C, then A and C are also likely to work well together. There may exist other stochastic structures that capture the same type of regularities, but the Salop structure does so in a particularly nice and tractable way. Note that we could alternatively let output depend positively on the difference between the workers, in order to capture a love of variety. To some extent this may be a matter of interpretation of what a good match is.

As indicated in the text, another representation which qualitatively captures the same properties are $n-1$ dimensional spheres in n-dimensional Euclidean space. With this model formulation, the distribution of distances of a new worker will be non-linear. More importantly, it may be convenient to choose a higher-dimensional location space if the number of workers in the team exceeds 3. In a two-dimensional space, it is not clear which of four workers are more peripheral. On a two-dimensional sphere, there are ways to deal with this, for example by defining closeness as the area of a circle on the sphere that contain all three locations. However, it is beyond the scope of this paper to explore these issues further.

We assume that wages are independent of match quality. As mentioned above, this is consistent with a competitive market where firms bid for ex ante identical workers prior to knowing the match quality. An alternative formulation would be to allow for bargaining, in which case part of the surplus from a good match would be allocated to the worker. This will give rise to a hold-up problem, if the firm pays the entire cost of replacing the worker and only gets a fraction less than one of the return in terms of a better match. The effect will be equivalent to reducing the circumference with a fraction equal to the workers' bargaining power, and can hence be easily captured within our framework. The effect will, naturally, be less replacement. In addition, if the firm is unable to extract the rents going to workers *ex ante* through a lower fixed wage, this rent will have to be dissipated in some other way, for instance through unemployment as in Shapiro and Stiglitz (1984) and Moen and Rosen (2006). Hence our model in this case may link worker replacement to the unemployment level. Furthermore, in the present version of the model, workers have no incentives to do on-the-job search, as wages are the same across firms. With wage bargaining, workers may have an incentive to search for a new job, and bargaining may therefore lead to on-the-job search.

Throughout we have assumed that the efficiency of a given team stays constant over time. Although a natural assumption as a starting point, one may think that the quality of a team may develop over time. As the workers get to know each other better, their ability to communicate and collaborate may improve. On the other hand, good relationships may sour over time. Introducing dynamics of team quality may lead to interesting hiring patterns. For instance, a firm that has been passive for a while may start a replacement frenzy if the relationship suddenly sours. This is on our agenda for future research.

7 Illustrative Simulations: Exploring the Mechanisms

We undertake simulations in order to explore the mechanisms inherent in the model. This gives a sense of the model's implications for worker turnover, firm age, firm value and the connections between them, revealing rich patterns. In particular, we examine the properties of the resulting firm value distributions and relate them to replacement policy. The dynamic evolution of these variables is due to both the random draw of workers and the firm's optimal replacement policy. The interaction of worker draws, exogenous shocks and firm policy is not trivial and generates non-normal firm value distributions. We explain the properties of these distributions, as expressed by their first four moments, in terms of the mechanisms of the model.

When simulating we look at the full model, with both endogenous and exogenous replacement and allowing for exogenous firm exit. As in the previous section, the value function is given by (18). Let $\bar{\beta}$ denote the pure time preference factor, where $\beta = \bar{\beta}(1 - s)$. This value function can be found by a fixed point algorithm. Appendix D provides full details. When simulating firms over time, we use the value function formulated above. We simulate 1000 firms over 30 periods, and repeat it 100 times to eliminate run-specific effects. In the benchmark case, we set $y := 1$; $c = 0.01$; $\bar{r} = 0.04$ (the pure discount rate), $= 0.1$; $s = 0.1$:

7.1 The Distribution of Firm Values

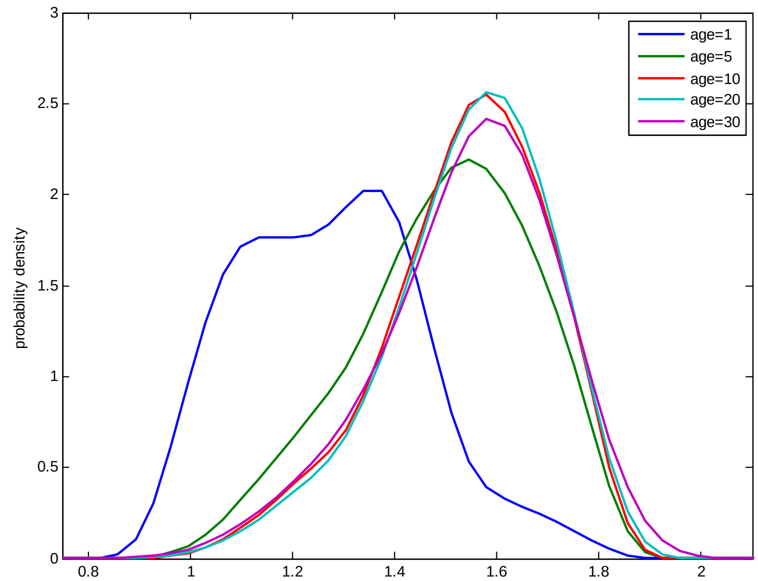

Figure 6a: Cross-sectional log …rm values, by age

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and the entry of new …rms, age 1 will be observed not only for all …rms in the …rst period, but also in all cases when a …rm exogenously left and was replaced by a new entrant. In this manner we gathered observations of values for all ages, from 1 to 30, and built the corresponding distributions.

Figure 6b: Moments of cross-sectional log …rms value, by age

The patterns re‡ect the pure process of convergence, disrupted from time to time by workers' exogenous exits, without the entry of new-born …rms. The value of the …rm grows with age as a result of team quality improvements, while the standard deviation is rather stable. As …rms mature, more of them enter the absorbing state, with relatively high values, and at the same time there are always unlucky …rms that do not manage to improve their teams su¢ ciently, or which have been hit by a forced separation shock. Therefore the distribution becomes more and more skewed over time. Excess kurtosis ‡uctuates.

These turnover dynamics of the model are very much in line with the …ndings in Haltiwanger, Jarmin and Miranda (2013), whereby, for U.S. …rms, both job creation and job destruction are high for young …rms and decline as …rms mature.

We run a regression of the simulation data to further study the connection between …rm value and …rm age. Here we look only at a simulated subsample of …rms which have survived until the 30th period. There have been 45 such …rms in our simulation. The estimated equation is:

$$\overline{\ln(V)}_t = c_0 + c_1 \ln(t) \quad (19)$$

where $\overline{\ln(V)}_t$ is the average logged value of …rms at age t, $t = 1; 2; :::; 30:$ The results are presented in Table 1:

Table 1

The Relation Between Firm Value and Age

Regression Results of Simulated Values

c_1	0.05
	(0.01)
c_0	1.37
	(0.02)
R^2	0.62

The coe¢ cients are highly signi…cant and imply a positive relation, illustrated below:

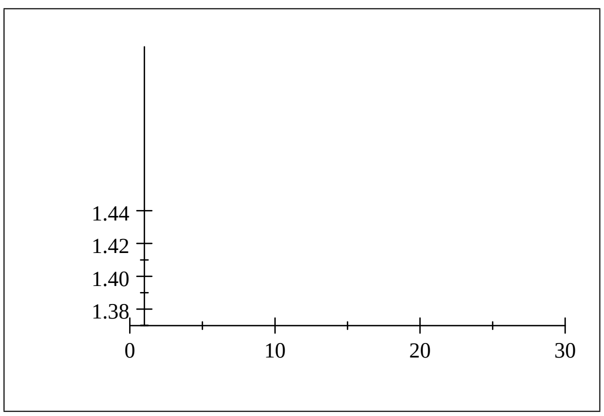

Figure 7: Predicted …rm value (logs) and …rm age

Figure 7 shows that overall, despite exogenous separation shocks, …rms tend to increase in value as they mature, due to the improvement of their teams' quality. This is in line with the …ndings of Haltiwanger, Lane and Spletzer (1999) whereby productivity rises with age for U.S. …rms in Census Bureau data, covering the period 1985-1996.

7.3 The Role of Model Parameters

The core parameters of the model at the benchmark are the worker replacement cost, $c = 0{:}01$; the annual rate of interest, $r = 0{:}04$; the exogenous

As …rms tend to stay with their current, randomly-drawn, teams, …rm values become more dispersed. Along the same lines, extreme values become relatively more frequent and excess kurtosis goes up. As inaction becomes optimal for so many …rms, …rms values become more concentrated above the mean. At the same time, in any period there are always unlucky …rms, which have just obtained a very bad team as a result of the or s shock. Hence skewness becomes more negative. The sensitivity to the interest rate is higher than to changes in replacement costs. Thus, under higher c or higher r the distribution has a longer left tail, lower mean, and fatter and longer tails relative to the benchmark.

(ii) Increases in the exogenous worker separation rate are illustrated in Figure 8b (and reported in rows 7-9 of Appendix Table E1).

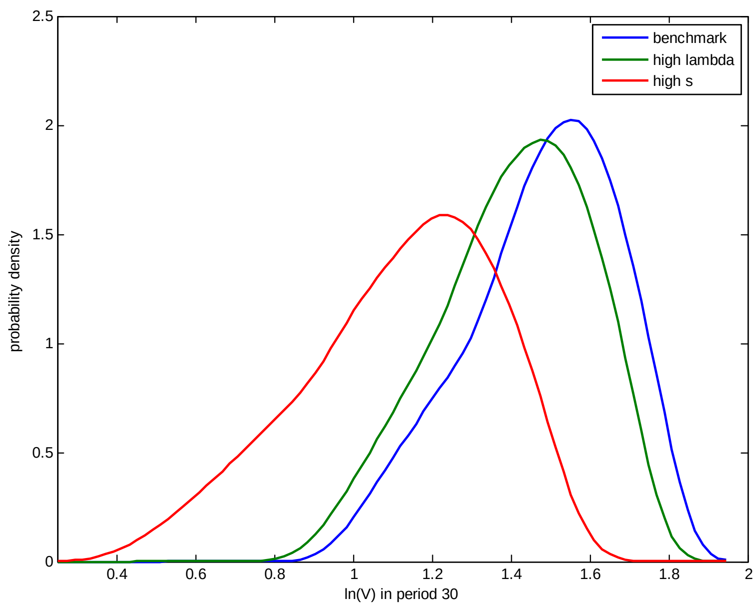

Figure 8b: e¤ects of and s

Increased separation depresses the mean value, slightly increases the co-e¢ cient of variation, make the skewness less negative and kurtosis more negative. The possibility of a worker's exogenous exit is a burden on the …rms, limiting their control over teams and the possibility to 6i6 sis more

8 Conclusions

The paper has characterized the …rm in its role as a coordinating device. Thus, output depends on the interactions between workers, with complementarities playing a key role. The paper has derived optimal policy, using a threshold on a state variable and allowing for endogenous hiring and …ring. Firm value emerges from optimal coordination done in this manner and ‡uctuates as the quality of the interaction between the workers changes. Simulations of the model generate non-normal …rm value distributions, with negative skewness and negative excess kurtosis. These moments re‡ect worker turnover dynamics, whereby a large mass of …rms is inactive in replacement, having attained good team formation, while exogenous replacement and …rm exit induce dispersion of …rms in the region of lower value. Future work will examine alternative production functions, learning and training processes, and wage-setting mechanisms within this set-up.

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9 Appendix A. Solution of the Cut-O¤

In this Appendix we show how to derive : We repeat the cut-o¤ equation for convenience

$$c + \frac{1}{2} + \frac{{}^{2}_{1}}{2} \quad {}_{1} \quad 2\bar{}_{2} = \frac{2\ {}_{1}\bar{}_{2} + 2\bar{}^{2}_{2}}{r} \qquad (20)$$

If $\bar{}_{2} = 0$, the left-hand side of (20) is strictly positive while the right-hand side is zero (since $_{1}$ 1=3 by construction). As $\bar{}_{2}$! 1 , the left-hand side goes to minus in…nity and the right-hand side to plus in…nity. Hence we know that the equation has a solution. Since the left-hand side is strictly decreasing and the right-hand side strictly increasing in $\bar{}_{2}$, we know that

10 Appendix B. Derivation of Equation (15)

Substituting (11) into (14) gives

$$\frac{y \quad 2({}_1 + \bar{}_2({}_1))}{r}(1 + r) + c \;=\; y \quad (\frac{1}{2} + {}_1 + \frac{{}_1^2}{2}) + \frac{({}_1 + 2\bar{}_2)(y \quad 2({}_1 + \overline{\quad}_2)) + 2\bar{}_2^2 + 2 {}_1\bar{}_2}{r} + (1 \quad {}_1 \quad 2\bar{}_2)\frac{y \quad 2({}_1 + \bar{}_2({}_1))}{r} \tag{23}$$

Collecting all terms containing $y \quad 2({}_1 + \bar{}_2({}_1))$ on the left-hand side gives

$$\frac{y \quad 2({}_1 + \bar{}_2({}_1))}{r}[1 + r \quad ({}_1 + 2 + 2$$

$$\bar{}_2 + {}_1 + \frac{{}_1^2}{2}) + \frac{2\bar{}_2^2 + 2 {}_1\bar{}_2}{r}$$

which simpli…es to

$$2({}_1 + \bar{}_2({}_1)) + c = (\frac{1}{2} + {}_1 + \frac{{}_1^2}{2}) + \frac{2\bar{}_2^2 + 2 {}_1\bar{}_2}{r} \tag{25}$$

Collecting terms gives

$$c + \frac{1}{2} + \frac{{}_1^2}{2} \quad {}_1 \quad 2\bar{}_2({}_1) = \frac{2\bar{}_2^2 + 2 {}_1\bar{}_2}{r} \tag{26}$$

which is equation (15).

11 Appendix C. Proof of Existence of Equilibrium

Define

$$\overline{V} \quad E^{j\ 1\ 2} V(\ _1;\ _2; 0; y; c)$$

Given our assumption that the firm always produces until it is destroyed, it follows that

$$E^{j\ 1\ 2} V(\ _1;\ _2; w; y; c) = \overline{V} \quad \frac{W}{r^0} \tag{27}$$

where $r^0 = r/(1+r)$ and where, as above, $W = 3w$. By assumption, $\overline{V} > 0$ (see below). It follows that there exists a unique W that solves the zero-profit condition given by

$$\overline{V} \quad \frac{W}{r^0} = K \tag{28}$$

The solution is given by $W = r^0(\overline{V} \quad K)$:

We will give conditions on parameters that ensure that $\overline{V} > 0$; and that

12 Appendix D. The Simulation Methodology

The entire simulation is run in Matlab with 100 iterations. In order to account for the variability of simulation output from iteration to iteration, or min(($_1$ + $_2$); 1 ($_1$ + $_2$)), with equal probabilities. In the general case, if there are two workers at a distance , and the third worker is drawn randomly, possible pairs in the following period may be of the following three types: (i) turns out to be the smaller distance (the third worker falls relatively far outside the arch), (ii) turns out to be the bigger distance (the third worker falls outside the arch, but relatively close) (iii) the third worker falls inside the arch, in which case the sum of the distances in the next period is . In the simulation we go over all possible pairs to identify the pairs that conform with (i)-(iii). Note that because all the distances are proportionate to 1/BINS_NUMBER, it is easy to identify the pairs of the type (iii) described above. This can be done for any , whether it is $_1$; $_2$ or min(($_1$ + $_2$); 1 ($_1$ + $_2$))

4. Having the guess V , and given that all possible pairs are equally probable, we are then able to calculate the expected values of the …rm when currently there are two workers at a distance . Call this value EV ( ). Then, if there is a …rm with three workers with distances ($_1$: $_2$), the expected value of voluntary replacement is EV ($_1$), and expected value of forced replacement is 1=3 EV ($_1$) + 1=3 EV ($_2$) + 1=3 EV (min(($_1$ + $_2$); 1 ($_1$ + $_2$))) :
[illegible] 2/21

5. According to $e_1; e_2$, using the calculations from previous section, we assign to each …rm the value and the optimal decision in the current period.

6. It is determined whether an exit shock hits. If it does, instead of the current distances of the …rm, a new triple is drawn in the next period. If it does not, it is determined whether a forced separation shock hits. If hits, a corresponding worker is replaced by a new draw and distances are recalculated in the next period. If it does not, and it is optimal not to replace, the distances are preserved for the …rm in the next period, as well as the value. If it is optimal to replace, the worst worker is replaced by a new one, distances are re-calculated in the next period, together with the value.

Steps 4-6 are repeated for each …rm over all periods.

As a result, we have a T by N matrix of …rm values. The whole process is iterated 100 times to eliminate run-speci…c e¤ects. We also record the events history, in a T by N matrix which assigns a value of 0 if a particular …rm was inactive in a particular period, 1 if it replaced voluntarily, 2 if it was forced to replace, and 3 if it was hit by an exit shock and ceased to exist from the next period on. We use this matrix to di¤erentiate …rms by states and to calculate …rms'ages.

13 Appendix E. Changes in Parameters

Table E1
The E¤ects of Changes in Parameters

	Parameters				Moments of ln(V) in period 30			
	c	r		s	mean	coef. of var.	skewness	excess kurtosis
1	0:01	0:04	0:1	0:1	1:46	0:13	0:47	0:40
2	0:05	10			1:45	0:14	0:55	0:28
3	0:10				1:44	0:16	0:68	0:06
4		0:01			1:60	0:10	0:39	0:53
5		0:04			1:46	0:13	0:47	0:40
6		0:10			1:15	0:20	0:72	0:02
7			0		1:73	0:11	0:67	0:04
8			0:05		1:58	0:12	0:58	0:27
9			0:15		1:46	0:13	0:41	0:48
10				0	2:82	0:02	0:21	0:52
11				0:05	1:86	0:07	0:41	0:40
12				0:15	1:09	0:22	0:53	0:32

0:32 10 0:52