The Ins and Outs of Selling Houses:
Understanding Housing-Market Volatility[*]

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Abstract

The housing market is subject to search frictions in buying and selling houses. This paper documents the role of inflows (new listings) and outflows (sales) in explaining the volatility and co-movement of housing-market variables. An `ins versus outs' decomposition shows that both inflows and outflows are quantitatively important in understanding fluctuations in houses for sale. The correlations between sales, prices, new listings, and time-to-sell are shown to be stable over time, while their correlations with houses for sale are found to be time varying. Using a housing-market model with endogenous inflows and outflows, a single persistent housing-demand shock can explain all the patterns of co-movement among variables except for houses for sale. Consistent with the data, the model does not predict there is an invariant structural relationship between houses for sale and other variables — the correlation depends on the source and persistence of shocks.

KEYWORDS: housing-market cyclicality; stocks and flows; search frictions.

1 Introduction

The importance of search frictions in buying and selling houses is widely acknowledged, with buyers and sellers spending considerable amounts of time searching. The essence of the search approach to markets is to understand how the stocks of buyers and sellers evolve through in ows and out ows. Applied to the labour market, this has been the subject of an extensive literature. However, for the housing market, there has been little work that aims to understand in ows and out ows jointly, especially with regard to cyclical uctuations.

This paper assembles a collection of stylized facts about the cyclical properties of a broad set of U.S. housing-market variables over the last three decades, including house prices and the key stocks and ows, comprising houses for sale, sales transactions, new listings, and the average time taken for houses to sell. A calibrated search-and-matching model with both endogenous in ows (new listings) and out ows (sales) is used to explain the empirical ndings.

One contribution of the paper is to document two novel facts. First, both in ows and out ows are quantitatively important in understanding housing-market volatility. This is shown using an `ins versus outs' decomposition of the type that has been applied to the labour market. Here, the stock of houses for sale is the equivalent of unemployment, the evolution of which depends on the difference between new listings and sales. The second novel fact is that houses for sale does not have a stable correlation with house prices, sales, or new listings, while correlations among all other pairs of variables remain stable. The correlations among prices, sales, and new listings are all positive, while the correlations of these with time-to-sell are all negative. On the other hand, while the correlation of houses for sales with time-to-sell has been positive throughout, the correlations of houses for sale with prices, sales, and new listings have changed from positive to negative in recent times.

A second contribution of this paper is to demonstrate two new quantitative results using a stochastic search-and-matching model with endogenous in ows and out ows. Central to the model is the idea of idiosyncratic match quality between a house and its owner, and the dynamics of the distribution of ongoing match quality. Decisions to buy houses are described by a cut-off rule whereby a sale occurs when a draw of new match quality is above a certain threshold. Individual match quality is a persistent variable, but is subject to occasional idiosyncratic shocks that degrade it. After such shocks, homeowners decide whether to move house, and the moving decision is also described by a cut-off rule for match quality. These decision processes give rise to an endogenous distribution of match quality across all homeowners.

The rst novel quantitative result is that a housing-demand shock can explain the patterns of co-movement among all variables with the exception of houses for sale. A housing-demand shock induces more moving and increases the supply of houses on the market. Hence, a single housing-demand shock replicates the three correlated, reduced-form shocks that have been used in the literature to match the behaviour of key housing-market variables.

Match quality plays a crucial role in the workings of the model and its ability to explain the

stylized facts with only one exogenous housing-demand shock. A positive demand shock raises the

contribution of this paper to the literature is in studying the role of new listings (in ows) alongside that of sales (out ows) in understanding the cyclical patterns of volatility and co-movement among housing-market variables.

Ngai and Sheedy (2020) construct a time series for the in ow rate to the housing market using a stock- ow accounting identity and show that it accounts for most of the long-run changes in the level of sales. The current paper uncovers two new facts about housing-market cyclicality. First, in ows are volatile, and uctuations in in ows have a clear pattern of cyclical co-movement with other housing-market variables. Changes in in ows are shown to be as important as out ows in accounting for uctuations in houses for sale. Second, in ows and out ows are positively correlated, and thus are associated with opposing effects on the number of houses for sale. This observation is closely related to the fact that correlations between houses for sale and other housing-market variables are not stable over time. In contrast, correlations among other pairs of variables are stable. This paper uses a stochastic version of the model of Ngai and Sheedy (2020) to highlight how the source and persistence of shocks affects the predicted responses of housing-market variables, which allows the model to replicate the changing correlation between houses for sale and prices that is seen over time.[2]

Smith (2020) also documents and studies the patterns of volatility and co-movement among new listings, sales, and houses for sale using data from the South Central Wisconsin Multiple Listing Service (SCWMLS) for Dane County between January 1997 to December 2007. The data in the current paper covers the whole of the U.S. and spans three decades, and one contribution here is in showing that the correlations between houses for sale and other variables have been time varying. While Smith (2020) focuses on generating hot and cold spells in sales in a stock- ow matching model with endogenous entry of sellers, the model in the current paper explores how moving deci-

Following Díaz and Jerez (2013), this paper uses real expenditures on `furnishings and durable household equipment' to calibrate a housing-demand shock. In their model, this demand shock on its own cannot generate the observed positive correlations between sales and prices, or between houses for sale and prices.[4] Here, this persistent demand shock *alone* successfully generates these two positive correlations. In the model, the endogeneity of moving decisions means that a housing-demand shock induces more moving, acting like a moving-rate shock, as well as increasing the supply of houses on the market, acting like a housing-supply shock. Thus, one housing-demand shock replicates the three correlated, reduced-form shocks needed in Díaz and Jerez (2013).[5]

Motivated by the positive correlation between houses for sale and prices documented by Díaz and Jerez (2013) prior to 2010, Gabrovski and Ortego-Marti (2019) argue that the housing market features an upward-sloping Beveridge curve, that is, a positive correlation between houses for sale and the number of buyers. Using an exogenous-moving model, they show that endogenous entry of houses and buyers can generate such a positive correlation. Here, the current paper shows that the endogenous moving decision of homeowners (related to `own-to-own' moves) naturally implies a positive correlation between houses for sale and the number of buyers in response to aggregate shocks. The quantitative analysis here demonstrates that a persistent demand shock can generate the observed positive correlation between houses for sale and prices s831-264TFy4that

2 The cyclical behaviour of housing-market variables

2.1 Volatility and co-movement

Standard deviations and correlation coefficients of sales transactions, house prices, new listings, houses for sale, and time-to-sell are shown in Table 1. The data have been transformed into natural logarithms to make the magnitudes of the cyclical fluctuations comparable across variables. Standard deviations of housing-market variables relative to sales transactions are also given in the table.

Table 1: Cyclical properties of housing-market variables

	Sales	Prices	New listings	Houses for sale	Time-to-sell
	Standard deviations				
	0.187	0.163	0.254	0.205	0.286
	Relative standard deviations				
Sales	1	0.872	1.36	1.10	1.53
	Correlation coefficients				
Sales	1				
Prices	0.720	1			
New listings	0.837	0.591	1		
Houses for sale	−0.062	0.220	−0.061	1	
Time-to-sell	−0.698	−0.312	−0.592	0.756	1

Notes: Calculated from natural logarithms of quarterly time series from 1991Q1 to 2019Q4. The original monthly data are seasonally adjusted by removing multiplicative month effects and then converted to a quarterly frequency. *Sources*: FHFA and NAR.

Díaz and Jerez (2013) document business-cycle facts for the housing market using data up to 2010.[11] The current paper builds on this earlier empirical work in two important ways. First of all, new listings are included as an additional variable, which is shown below to be quantitatively important for understanding cyclical fluctuations in the housing market. Second, this paper assembles data on sales transactions, the number of houses for sale, and average time-to-sell from the same source rather than the three different sources used by Díaz and Jerez (2013). More specifically, in Díaz and Jerez (2013), sales data are taken from NAR as here, time-to-sell is measured only for newly constructed houses (`New Residential Sales' from the U.S. Census Bureau), and data on houses for sale come from the `vacant for sale' measure provided by the U.S. Census Bureau Housing Vacancy Survey. Note that this `vacant for sale' data include only a small fraction of the houses that are actually for sale because houses that are occupied but available for sale are excluded. Vacant houses are only around 11% of all single-family homes sold.[13]

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[11]A table directly comparable to D

As is well known in the literature, Table 1 shows house prices and sales positively co-move with a correlation coefficient of 0.72, there is a negative correlation between time-to-sell and sales with correlation coefficient -0.70, and the volume of sales transactions is highly volatile. In addition to these familiar facts, Table 1 reveals that new listings are as volatile as sales.[13] New listings positively co-move with sales and prices with correlation coefficients 0.84 and 0.59 respectively, and negatively co-move with time-to-sell with correlation coefficient -0.59. Finally, houses for sale are uncorrelated with sales volume and new listings, but positively correlated with prices and time-to-sell. These last two positive correlations are also documented by Díaz and Jerez (2013) using `vacant for sale' as the measure of houses for sale.

2.2 The ins and outs of houses for sale

In studying the housing market as a market subject to search frictions, the stock of houses for sale is analogous to unemployment in the labour market. As in the labour literature, it is possible to understand fluctuations in houses for sale in terms of changes in the rates of inflows and outflows to and from the housing market. A higher inflow rate (more new listings) increases houses for sale; a higher outflow rate (more sales) decreases houses for sale. Methodologically, this section follows the `ins versus outs' decompositions of unemployment fluctuations (Petrongolo and Pissarides, 2008, Fujita and Ramey, 2009, Elsby, Hobijn and Şahin, 2013) to investigate the source of cyclical fluctuations in houses for sale using the same techniques as have been applied in research on labour markets.

The inflow and outflow rates in the housing market are respectively the rate at which houses are listed for sale and the rate at which they are subsequently sold. The sales rate $s_t = S_t/U_t$ is measured as the ratio of sales transactions S_t to houses for sale U_t. This is the inverse of the time-to-sell measure $T_t = U_t/S_t$ introduced earlier. The listing rate n_t is the ratio of the number of new listings N_t to the number of houses not currently listed for sale, that is, the difference between the total housing stock K and houses for sale U_t. The formula for the listing rate is $n_t = N_t/(K - U_t)$. In practice, since the total housing stock K far exceeds the number of houses for sale, the listing rate n_t is close to being proportional to new listings N_t.

The inflow and outflow rates n_t and s_t are calculated with the data from NAR on sales and inventories described earlier. These data are used to construct series for new listings N_t using the stock-flow accounting identity, and the measure U_t of houses for sale. In calculating the inflow rate n_t, though not the outflow rate s_t, a measure of the total housing stock K is also needed. However, the main effect of K is on the average level of the inflow rate n_t, not the cyclical fluctuations that are the focus of this paper.[14] It turns out to make little difference to the following inflow-outflow decomposition exactly what value of K is used within some reasonable range. For the purposes of

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[13]This is consistent with Bachmann and Cooper (2014), who show that housing turnover is volatile using data on flows within the owner-occupied segment of the housing market obtained from the Panel Study of Income Dynamics.

[14]The total housing stock K is treated as a constant here because high-frequency data are not available. The role of a time trend in the housing stock in explaining long-run changes in sales volume is explored in

this study, the total housing stock should measure all houses that are either for sale or might be put up for sale, and the number should be consistent with the sales and inventories data from NAR for existing single-family homes. Using information from the U.S. Census Bureau American Housing Survey and New Residential Construction data, the total housing stock K is set to be 50 million as an approximation.

Figure 1 plots the quarterly in ow and out ow rates. These are used to perform an in ows-out ows decomposition of uctuations in houses for sale $u_t = U_t / K$ as a fraction of the total housing stock. Using the stock- ow accounting identity, the law of motion for u_t is approximately

$$\text{D}u_t \quad n_t(1 - u_t) - s_t u_t; \tag{1}$$

where $n_t(1 - u_t)$ is the in ow and $s_t u_t$ is the out ow, both relative to the total stock of houses.[15]

Figure 1: In ow and out ow rates in the housing market
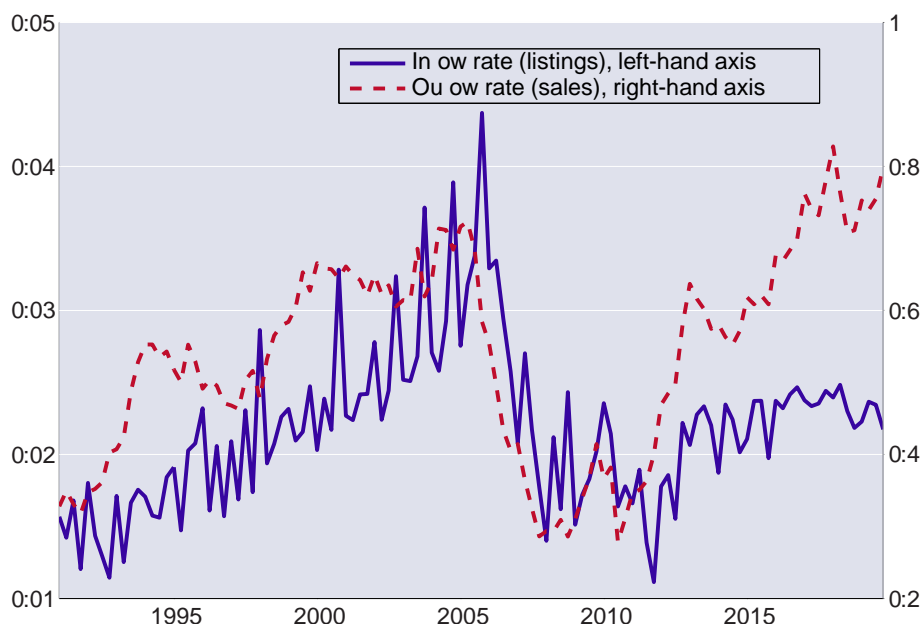
Notes: Quarterly time series from 1991Q1 to 2019Q4. The original monthly data are seasonally adjusted by removing multiplicative month effects and then converted to a quarterly frequency.
Source: NAR.

Several commonly used methods for performing the decomposition are based on the time-varying

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[15]A re nement of this equation uses estimates of the continuous-time in ow and out ow rates to account explicitly for ows occurring within time periods. This is done in Petrongolo and Pissarides (2008), for example. Here, note that houses for sale u_t is calculated using an average of beginning-of-period and end-of-period inventory, which partially addresses this issue. In practice, there is no signi cant effect on the results presented below if continuous-time rates n_t and s_t are calculated using the method in Petrongolo and Pissarides (2008).

steady state u_t of the fraction of houses for sale, that is, the value of u_t such that $Du_t = 0$ in (1):

$$u_t = \frac{n_t}{s_t + n_t};$$ (2)

The argument for focusing on u_t instead of the actual u_t is that convergence to the steady state is expected to be rapid: the rate of convergence is the sum of the inflow and outflow rates. It is implicitly assumed that u_t is close enough to u_t to study the contributions of inflow and outflow rates to fluctuations in u_t through the effects of n_t and s_t on u_t in (2).

Fujita and Ramey (2009) note that changes in $\log u_t$ over time are approximately given by

$$Dlog\, u_t \quad (1 - u_t)(Dlog\, n_t - Dlog\, s_t);$$ (3)

where $Dlog\, n_t$ and $Dlog\, s_t$ are the changes in log inflow (listings) and outflow (sales) rates. From this

Table 2: In ow-out ow decompositions of uctuations in houses for sale

Method	New listings (ϕ_n)	Sales (ϕ_s)

Figure 2: Rolling correlations of housing-market variables

Correlations with houses for sale

Correlations with sales

Notes Correlation coef cients in 10-year windows are calculated using seasonally adjusted quarterly time series in logarithms. The date on the horizontal axis gives the mid-point of the 10-year window. Sources FHFA and NAR.

from Figure 2 that the correlation coef cients of houses for sales with sales, prices, and new listings change drastically from positive and negative across the two sub-samples, while the other correlation coef cients have stable signs. These ndings provide evidence that there is no invariant structural relationship between houses for sale and prices, new listings, and sales. As shown later in section 4 using a calibrated search-and-matching model, the changing sign of these correlation coef cients can be explained through changes in the persistence and nature of the shocks affecting the housing market.

Finally, since all the earlier analysis of the behaviour of new listings was based on numbers imputed from a stock- ow accounting identity, directly measured data on new listings from Red n are used as a robustness check on the empirical ndings in Table 1 and Table 3. Red n data on new listings, sales transactions, inventories, prices, and days on the market are available monthly from February 2012.[19] The Red n house-price series is divided by the PCE price index to obtain

[19]Red n is a real-estate brokerage with direct access to data from local Multiple Listing Services (MLS). Methodology

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Table 3: Cyclical properties of variables in the 1991–2009 and 2010–2019 sub-samples

	Sales		Prices		New listings		Houses for sale		Time-to-sell	
Standard deviations										
	0.22	0.10	0.17	0.11	0.29	0.18	0.20	0.19	0.27	0.29
18ime-Relative standard deviations										
Sales	1	079	1.94081229552 T0.880.585 -14.446 Td [11.95- Td9.312 0 [1							

factor $b_t = e^{-\Delta\,r_t}$. Expectations conditional on information available at time t are denoted by $E_t[\cdot]$.

3.1 Behaviour of buyers and sellers

Search frictions The housing market is subject to search frictions. First, it is time-consuming and costly for buyers and sellers to arrange viewings of houses. Let u_t denote the measure of houses listed for sale and b_t the measure of buyers. Each buyer and each house can have at most one viewing in the time interval $[t, t + \Delta)$.[21] For houses, this event has Poisson arrival rate $M(u_t, b_t)/u_t$, where $M(u, b)$ is a constant-returns meeting function (noting that not all viewings will lead to matches). For buyers, the corresponding arrival rate is $M(u_t, b_t)/b_t$. During this process of search, buyers incur flow search costs F per interval of time Δ.

Given the unit measure of houses, there are $1 - u$ houses that are matched in the sense of being occupied by a household. As there is also a unit measure of households, there must be u households not matched with a house, and thus in the market to buy. This means the measures of buyers and sellers are the same ($b_t = u_t$). Given that the function $M(u, b)$ features constant returns to scale, the arrival rates of viewings for buyers and sellers are then both equal to $m = M(1, 1)$. This m summarizes all that needs to be known about the frictions in locating houses to view.

to an offer to buy, the gain is the transaction price, and the loss is the option value of continuing to search, namely $b_t E_t V_{t+t}$, where V_t is the value of owning a house for sale. Finally, the buyer and seller face a combined transaction cost C. The total surplus $S_t(e)$ resulting from a transaction with match quality e at date t is given by

$$S_t(e) = H_t(e) - b_t E_t J_{t+t} - C; \quad \text{where } J_t = B_t + V_t;$$ (6)

with J_t denoting the combined value of being a buyer and having a house for sale. Since the value function $H_t(e)$ is increasing in e, transactions occur if match quality e is no lower than a threshold y_t, de ned by S_t (y) ncestadveb7(f)10(ace)-276Tf 91.9-297er

and house being:

where $a = e^{-\delta t}$ is the probability that no idiosyncratic shock is received during $[t, t + \tau)$.

Listing decisions Following the arrival of idiosyncratic shocks, homeowners decide whether to list their homes for sale on the market or not. The value function $H_t(e)$ for an owner-occupier is determined by the Bellman equation

$$H_t(e) = \tau e q_t + a b_t E_t \max\{H_{t+\tau}(e) - \tau D_i J_{t+\tau} - z\}$$
$$+ (1 - a) b_t E_t \max\{H_{t+\tau}(\delta e) - \tau D_i J_{t+\tau}\};$$

where z is an inconvenience cost of moving faced only by those who do not experience an idiosyncratic shock. This cost represents the inertia of families to remain in the same house. It is assumed

3.3 Solving the model

In the case of no aggregate shocks ($h_{q,t} = 0$ and $h_{r,t} = 0$ for all t, so $q_t = 1$ and $r_t = r$ in 18), the model becomes a discrete-time version of Ngai and Sheedy (2020). With aggregate shocks, the solution of the model for aggregate variables is obtained approximately using a first-order perturbation (log linearization) around the deterministic steady state ($s_q = 0$ and $s_r = 0$). The well-known problem of non-differentiability in models of endogenous `lumpy' adjustments — here, the decision to list a house for sale — is overcome given two parameter restrictions, while the Pareto distribution of new match quality significantly reduces the size of the model's state space.

Large idiosyncratic shocks First, idiosyncratic shocks are assumed to be large (in 15 sufficiently far below 1) relative to aggregate shocks (the standard deviations s_q and s_r in 18 are sufficiently small), and large relative to the difference between the transaction and moving thresholds y_t and x_t, which depends mainly on the transaction cost C. Second, the inconvenience cost z faced by those who do not receive an idiosyncratic shock is large relative to the size of the aggregate shocks.

Intuitively, the role of relatively large idiosyncratic shocks is illustrated in Figure 3, which shows the distribution of e for existing matches, which was previously truncated at some point w. The left panel shows the case where no idiosyncratic shock occurs. Without the cost z, the endogenous moving decision would imply a `kinked' response of the overall number of homeowners who move. The idea is that if the moving threshold falls due to an aggregate shock then there is no change in the number of homeowners who move, unlike the case where the moving threshold rises. The right panel shows the case where idiosyncratic shocks are large relative to changes in the moving thresholds due to aggregate shocks. In that case there is no problem of non-differentiability. When no idiosyncratic shock is received, the non-differentiability problem is avoided by a sufficiently large cost z.

Figure 3: Differentiability and idiosyncratic shocks
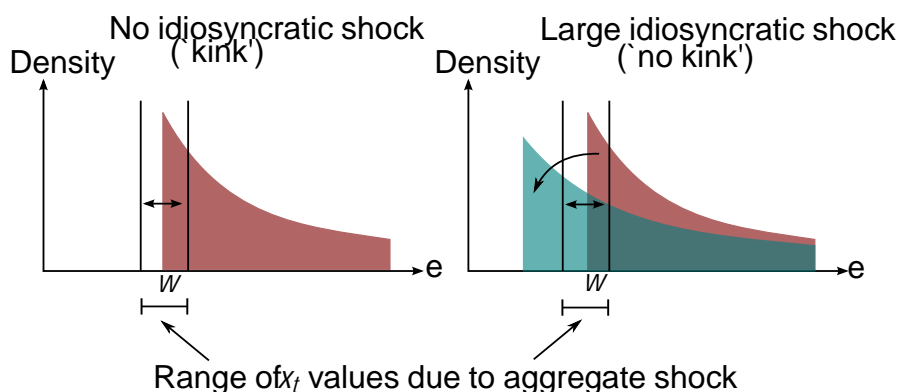
The magnitude of fluctuations in the transaction and moving thresholds y_t and x_t is small relative to the changes in e brought about by idiosyncratic shocks when the standard deviations s_q and s_r

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from (18

idiosyncratic shock but who decide not to move.

Aggregating listing decisions All matches begin as draws from the distribution of match quality $e \sim$ Pareto$(1, l)$. Surviving matches that receive an idiosyncratic shock during the interval $(t-1, t]$ can be characterized by their initial match quality e, their vintage v, where $v \in \{1, 2, 3, \ldots\}$ denotes the number of discrete time periods since the match formed, and the number $q \in \{0, 1, \ldots, v-1\}$ of previous idiosyncratic shocks that have occurred. At date t immediately after an idiosyncratic shock, current match quality is now $e^0 = d^{q+1}e$ given original match quality e. A match survives the current shock only if $e^0 \geq x_t$, or equivalently $e \geq x_t = d^{q+1}$ in terms of its original match quality.

Matches with vintage v at date t originate from the measure $m u_{t-\tau_v}$ of past viewings. Depending on the timing of the realization of past idiosyncratic shocks, matches with vintage v by date t and q previous shocks are those that remain after truncating the distribution of original match quality e to the left at various points. These truncations occur with the first transaction decision $(e \geq y_{t-\tau_v})$ and subsequent moving decisions $(e \geq x_{t-\tau_i} = d^{j+1}$ for some $i = 1, \ldots, v-1$ and some $j = 0, \ldots, q)$. Let $G_{t, v, q}(w)$ denote the distribution function of the truncation points w of the original distribution of match quality for the cohort of vintage v by date t with q previous idiosyncratic shocks.

The properties of the Pareto distribution imply that the distribution of e conditional on $e \geq w$ is Pareto(w, l) with the original shape parameter l. If $x_t = d^{q+1} \leq w$ for all w in the distribution $G_{t, v, q}(w)$, that is, $G_{t, v, q}(x_t = d^{q+1}) = 1$, then the probability of a match surviving the current shock conditional on any particular w and the original match having $e \geq w$ is $P[e \geq x_t = d^{q+1} | e \geq w] = (x_t = (d^{q+1}w))^{-l}$. Since the possible truncation points are $w = y_{t-\tau_v}$ or $w = x_{t-\tau_i} = d^{j+1}$ for some $i \in \{1, \ldots, v-1\}$ and $j \in \{0, \ldots, q\}$, for a given range of fluctuations in the thresholds y_t and x_t, this formula is valid if d is sufficiently far below 1 because it implies $dx_t < x_{t^0}$ and $dy_t < x_{t^0}$ for all t and t^0.

Conditional on vintage v, the independence of successive idiosyncratic shocks implies $q \sim$ Binomial$(v-1, 1-a)$, where $v-1$ is the maximum number of previous shocks and $1-a$ is the probability of each shock. With original match quality of the mass $m u_{t-\tau_v}$ of viewings previously truncated to the left of $e = w$, a fraction w^{-l} of the initial draws of e survived as matches up to the point where the current idiosyncratic shock occurs. Putting together these observations, the measure of matches receiving and surviving an idiosyncratic shock in the interval $(t-1, t]$ is

$$(1-a)\sum_{v=1}^{\infty} m u_{t-\tau_v} \sum_{q=0}^{v-1} \frac{(v-1)!}{q!(v-1-q)!}(1-a)^q a^{v-1-q} \int_w \left(\frac{x_t}{d^{q+1}w}\right)^{-l} w^{-l}\, dG_{t, v, q}(w)$$

$$= m(1-a)d^l x_t^{-l} \sum_{v=1}^{\infty} u_{t-\tau_v} \sum_{q=0}^{v-1} \frac{(v-1)!}{q!(v-1-q)!}\left((1-a)d^l\right)^q a^{v-1-q} \int_w dG_{t, v, q}(w)$$

$$= m(1-a)d^l x_t^{-l} \sum_{v=1}^{\infty}\left(a+(1-a)d^l\right)^{v-1} u_{t-\tau_v}.$$

The first line uses the probability $_{v-1}C_q(1-a)^q a^{v-1-q}$ of drawing q from the binomial distribution,

Table 5: Calibrated parameters

Parameter description	Notation	Value	Continuous-time rate
Length of a discrete time period	t	$1/52$	
Discount factor (steady state)	b	0.9989	$r = 0.057$
Probability of no idiosyncratic shock	a	0.9978	$a = 0.116$
Size of shocks	d	0.903	
Distribution of new match quality	l	17.6	
Probability of a viewing	m	0.2994	$m = 18.5$
Total transaction costs	C	0.611	
Flow search costs	F	0.153	
Flow maintenance costs	D	0.275	
Share of total transaction costs directly borne by seller	k	$1/3$	
Bargaining power of sellers	w	$1/2$	

Notes: These parameters are taken from the calibrated continuous-time model in Ngai and Sheedy (2020), with discrete-time equivalents $b = e^{-rt}$, $a = e^{-at}$, and $m = 1 - e^{-mt}$ calculated for the weekly length of a discrete time period ($t = 1/52$).

Aggregate shocks There are aggregate shocks to housing demand q and the discount rate r in the model. The empirical counterparts to these variables are taken to be real expenditures on furnishings and durable household equipment (as is also done by Díaz and Jerez, 2013) and the short-term real interest rate. A formal justification is provided in Appendix A.13 of Ngai and Sheedy (2020). Intuitively, housing demand q_t appears in households' utility multiplicatively with match quality e

4.2   A single housing-demand shock

To begin with, this section explores how much of the patterns of cyclical uctuations can be explained by a single housing-demand shock.

sale depends on the difference between the changes in listings and sales. In the case shown here, listings rise by slightly more than transactions initially, so houses for sale also increase slightly. More generally, the persistence of the demand shock affects the relative size of the listings and sales responses, and thus there is not an unambiguous prediction from the model about whether houses for sale will rise or fall. Later in section 4.4, the model is simulated using the stochastic properties of housing demand in two sub-samples to illustrate this point.

Table 6 reports the model-implied standard deviations and correlation coefficients among the housing-market variables, assuming for now all fluctuations are driven by demand shocks. Compared to the data presented in Table 1, the model with only a housing-demand shock matches well the positive correlations of sales with prices and new listings, the positive correlations of prices with new listings and houses for sale, and the negative correlations of time-to-sell with sales, prices, and new listings. The model also does reasonably well in generating a fair amount of volatility in the housing market, though with only one shock, it is perhaps not surprising that it does not completely account for all the volatility seen in the data. As a point of comparison with Díaz and Jerez (2013), here, only a demand shock matching the stochastic properties of equipment expenditure is used, whereas they have to add correlated supply and moving-rate shocks calibrated to match the time-series properties of sales and houses for sale. By construction, they match the standard deviation of sales and houses for sale.

Table 6: Model-predicted cyclicality of variables with only shocks to housing demand

Demand	Sales	Prices	New listings	Houses for sale	Time-to-sell
			Standard deviations		
0.097	0.067	0.081	0.067	0.003	0.065
			Relative standard deviations		
Sales	1	1.20	1.00	0.045	0.963
			Correlation coefficients		
Sales	1				
Prices	0.999	1			
New listings	1.00	0.999	1		
Houses for sale	0.954	0.950	0.954	1	
Time-to-sell	0.999	1.00	0.999	0.950	1

Notes Simulated moments of the theoretical model with $\beta = 0.9873^{1=13}$, $s_q = \sqrt{1 - f_q^2} \cdot 0.0965$, and $s_r = 0$ so that only housing-demand shocks occur.

Match quality plays a crucial role in the workings of the model and its ability to match many of

is viewed by a potential buyer, new match quality is drawn from a probability distribution, and there is a transaction threshold at which the buyer is willing to trade. A positive housing-demand shock raises the total surplus from a transaction and thus increases both the willingness to trade and the price paid, which gives rise to a positive correlation between sales and prices. This correlation would be negative in the absence of a distribution of new match quality, as found in the model of Díaz and Jerez (2013) when there is only a demand shock.

On the other hand, the equilibrium distribution of match quality among existing homeowners is key to explaining the positive correlation between sales and new listings. Homeowners' match quality is a persistent variable subject to occasional idiosyncratic shocks. At any point in time, there is an endogenous distribution of match quality across existing homeowners, and a moving

another important factor for such an investment decision.

Figure 5 shows the impulse responses of housing-market variables to a negative unit (1 percentage point) shock to the real interest rate r_t. A fall in the real interest rate lowers the discount rate applied to housing flow values, increasing the total surplus from a transaction and raising the price paid. A lower interest rate increases homeowners' incentives to invest in better match quality because it raises the relative importance of future payoffs compared to current costs. Hence, a lower interest rate has a similar effect on prices and new listings as does a positive demand shock.

Figure 5: Impulse responses of variables to an interest-rate shock
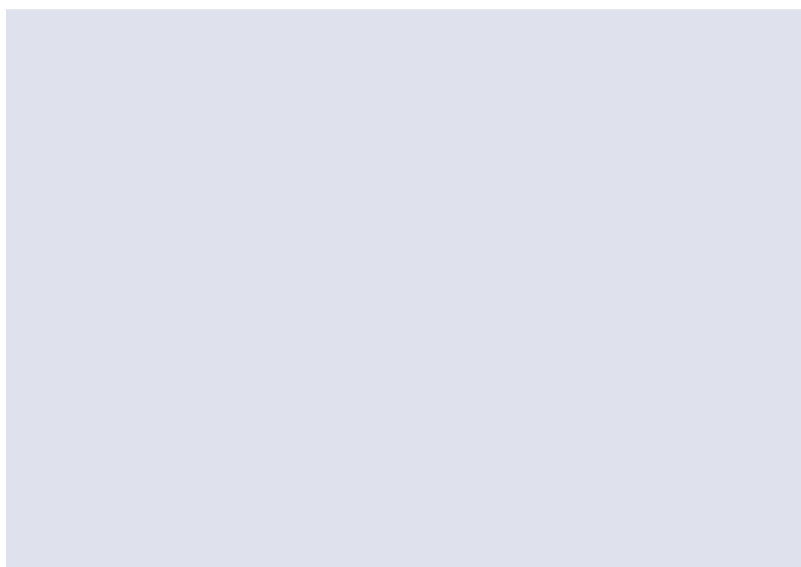
Notes: The interest-rate shock has persistence given by $0.933^{t=13}$.

However, compared to a positive demand shock, a lower interest rate has the opposite effect on time-to-sell. Since the lower interest rate increases the relative importance of future payoffs, it raises the returns to searching, leading to longer time-to-sell. This subdues the initial rise in sales, and with a greater gap between the impulse responses of new listings and sales, the increase in houses for sale is much larger. The different behaviour of time-to-sell for the interest-rate shock can thus explain a positive correlation between houses for sale and time-to-sell, as is found empirically. This exercise reveals that the source of shocks is important in understanding housing-market cyclicality.

Table 7 reports the model-implied standard deviations and correlation coefficients of housing-market variables when both independent demand and interest-rate shocks occur. Compared to Table 6, introducing an additional interest-rate shock increases the volatility of all variables, but more so for houses for sale and less so for prices, which improves the predicted relative standard deviations of these two variables. The correlation between houses for sale and time-to-sell becomes positive overall. Adding the interest-rate shock also moves the correlation coefficients of houses for

Table 8: Model-predicted cyclicality with both shocks in two sub-sample periods

	Sales	Prices	New	Houses	Time-

Figure 6: Impulse responses to a less persistent demand shock

Notes: The housing-demand shock has persistence given by

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A   Appendices

A.1   Volatility and co-movement with detrended data

To compare the cyclical properties of the data with those found by Díaz and Jerez (2013), the seasonally adjusted quarterly time series in natural logarithms are detrended using the Hodrick-Prescott filter (with smoothing parameter 1600). The results are displayed in Table 9.

Table 9: Cyclical properties of HP-filtered housing-market variables

	Sales	Prices	New listings	Houses for sale	Time-to-sell
	Standard deviations				
	0.067	0.025	0.15	0.073	0.110
	Relative standard deviations				
Sales	1	0.366	2.24	1.09	1.63
	Correlation coefficients				
Sales	1				
Prices	0.399	1			
New listings	0.456	0.289	1		
Houses for sale	−0.215	0.121	−0.120	1	
Time-to-sell	−0.757	−0.164	−0.360	0.800	1

Notes: Calculated from HP-filtered (smoothing parameter 1600) natural logarithms of quarterly time series from 1991Q1 to 2019Q4. The original monthly data are seasonally adjusted by removing multiplicative month effects and then converted to a quarterly frequency.
Sources: FHFA and NAR.

The statistics related to sales, prices, houses for sale, and time-to-sell are similar to those reported in Díaz and Jerez (2013). In addition to the differences in the measurement of houses for sale and time-to-sell discussed in section 2, note also that while the time series here all cover the period 1991Q1–2019Q4, Table 1 of Díaz and Jerez (2013) uses different time periods for different variables. For example, their measure of sales starts from 1968, but the price series starts from 1975 or from 1990.

The overall cyclical patterns are broadly consistent with those presented in Table 1. The levels of the standard deviations are lower, but the ranking of the relative standard deviation is similar to before. To highlight a few differences in the correlation coefficients compared to Table 1, the positive correlations between house prices and sales, new listings and sales, and new listings and prices are all weaker with correlation coefficients of 0.399, 0.456, and 0.289, respectively. The negative correlations of time-to-sell with prices and new listings are also weaker with correlation coefficients of −0.164 and −0.360, respectively.

Figure 7 reports rolling correlations in 10-year windows of HP-filtered data on housing-market variables. It displays the same pattern seen Figure 2 where the correlations of houses for sale with sales, prices, and new listings change sign over time, while the correlation of sales with prices, new listings, and time-to-sell are stable.

Figure 7: Rolling correlations of HP- ltered housing-market variables

Correlations with houses for sale

Correlations with sales

A.2 Characterizing aggregate dynamics with a nite number of variables

This section derives a set of equations in a nite number of variables that characterizes the aggregate dynamics
of the housing market. Under the assumptions made in section 3.3, the idiosyncratic shock is suf cient large
(d is suf ciently far below 1) that $dx_t < x_{\theta}$ and $dy_t < x_{\theta}$ for all t and t^{θ}. Consequently, there exists a threshold
x, which lies above y_t and x_t for all t, such that $de < x_{t+t}$ for any e x. Since $H_{t+t}(e)$ is increasing in e, it
follows using (17) that $H_{t+t}(de) - tD < J_{t+t}$ for all e x and thus max$fH_{t+t}(de) - tD; J_{t+t}g = J_{t+t}$. The
Bellman equation (16) for e x becomes

$$H_t(e) = teq_t + ab_tE_t[H_{t+t}(e) - tD] + (1-a)b_tE_tJ_{t+t}:$$ (A.1)

Differentiating with respect to e gives $H_t^{\emptyset}(e) = tq_t + ab_tE_tH_{t+t}^{\emptyset}(e)$, which can be iterated forwards to deduce:

$$H_t^{\emptyset}(e) = Q_t; \quad \text{where } Q_t = t\,E_t\,\,q_t + ab_tq_{t+1} + a^2b_tb_{t+1}q_{t+2} + \cdots :$$

The variable Q_t depends only on the exogenous variables q_t and b_t and satis es the expectational difference
equation

$$Q_t = tq_t + ab_tE_tQ_{t+t}:$$ (A.2)

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Since $H_t^0(e)$ is independent of e for $e \geq x$, it follows that $H_t(e)$ is linear for $e \in [$

$$H_t(e) = L_t + Q_t e; \tag{A.3}$$

for some variable L_t independent of e. Substituting back into (A.1) implies $L_t +$ $\quad$ $[L_{t+t} +$ $Q_{t+t} e - t D] + (1 - a)b_t E_t J_{t+t}$, and then replacing Q_t using (A.2) yields

$$L_t = ab_t E_t L_{t+t} - ab_t t D + (1 - a)b_t E_t J_{t+t};$$

Since $x_t < x$, equation (A.3) can be evaluated at $e = x_t$, hence H

that de nes the moving threshold, it follows that $L_t = J_t + Q_t \quad t$

variable $e^0 = de$, and noting $dz_t < x_{t+\tau}$ because $z_t < x$:

$$\int_{e=z_t}^{\infty} \lambda e^{-(\lambda+1)} \max\{H_{t+\tau}(de) - H_{t+\tau}(x_{t+\tau}); 0\} de$$

$$= d^{\lambda} \int_{e^0 = dz_t}^{\infty} \lambda (e^0)^{-(\lambda+1)} \max\left\{H_{t+\tau}(e^0) - H_{t+\tau}(x_{t+\tau}); 0\right\} de^0 = d^{\lambda} \int_{e^0 = dz_t}^{x_{t+\tau}} \lambda (e^0)^{-(\lambda+1)} 0 \, de^0$$

$$+ d^{\lambda} \int_{e^0 = x_{t+\tau}}^{\infty} \lambda (e^0)^{-(\lambda+1)} \left[H_{t+\tau}(e^0) - H_{t+\tau}(x_{t+\tau})\right] de^0 = d^{\lambda} Y_{t+\tau}(x_{t+\tau}); \quad (A.9)$$

which uses $H_{t+\tau}(e^0) < H_{t+\tau}(x_{t+\tau})$ for $e^0 < x_{t+\tau}$, and the definition of $Y_t(z_t)$ from (A.7). Note also:

$$\int_{e=z_t}^{\infty} \lambda e^{-(\lambda+1)} de = z_t^{-\lambda}; \quad \text{and} \quad \int_{e=z_t}^{\infty} \lambda e^{-(\lambda+1)}(e - z_t) de = \frac{z_t^{1-\lambda}}{\lambda - 1}: \quad (A.10)$$

Since $z_t < x$ and $z_{t+\tau} < x$, it follows from (A.3) that $H_{t+\tau}(e) - H_{t+\tau}(z_{t+\tau}) = Q_{t+\tau}(e - z_{t+\tau})$ for all e between z_t and $z_{t+\tau}$. Breaking up the range of integration in the following equations and using the definition $Y_t(z_t)$ from (A.7) leads to

$$\int_{e=z_t}^{\infty} \lambda e^{-(\lambda+1)} (H_{t+\tau}(e) - H_{t+\tau}(z_{t+\tau})) de = \int_{e=z_t}^{z_{t+\tau}} \lambda e^{-(\lambda+1)} (H_{t+\tau}(e) - H_{t+\tau}(z_{t+\tau})) de$$

$$+ \int_{e=z_{t+\tau}}^{\infty} \lambda e^{-(\lambda+1)} (H_{t+\tau}(e) - H_{t+\tau}(z_{t+\tau})) de = Y_{t+\tau}(z_{t+\tau}) + Q_{t+\tau} \int_{e=z_t}^{z_{t+\tau}} \lambda e^{-(\lambda+1)}(e - z_{t+\tau}) de$$

$$= Y_{t+\tau}(z_{t+\tau}) + Q_{t+\tau} \left[\frac{\lambda}{\lambda - 1} \left(z_t^{1-\lambda} - z_{t+\tau}^{1-\lambda} \right) + z_{t+\tau} \left(z_{t+\tau}^{-\lambda} - z_t^{-\lambda} \right) \right]: \quad (A.11)$$

Note also that $H_{t+\tau}(z_{t+\tau}) - H_{t+\tau}(z_t) = Q_{t+\tau}(z_{t+\tau} - z_t)$ using (A.3). By combining equations (A.7), (A.8), (A.9), (A.10), and (A.11), the following result holds for all $z_t < x$:

$$Y_t(z_t) = \tau q_t \frac{z_t^{1-\lambda}}{\lambda - 1} + a b_t E_t Y_{t+\tau}(z_{t+\tau}) + (1-a) d^{\lambda} b_t E_t Y_{t+\tau}(x_{t+\tau})$$

$$+ a b_t E_t \left[(z_{t+\tau} - z_t) z_t^{-\lambda} + \frac{\lambda}{\lambda - 1} \left(z_t^{1-\lambda} - z_{t+\tau}^{1-\lambda} \right) + z_{t+\tau} \left(z_{t+\tau}^{-\lambda} - z_t^{-\lambda} \right) \right] Q_{t+\tau}$$

$$= \tau q_t \frac{z_t^{1-\lambda}}{\lambda - 1} + a b_t E_t Y_{t+\tau}(z_{t+\tau}) + (1-a) d^{\lambda} b_t E_t Y_{t+\tau}(x_{t+\tau}) + a b \left(1 - a \right) \tau_t \lambda$$

variables from (A.13):

$$c_t - \frac{Q_t x_t^{1-\iota}}{\iota - 1} = \alpha \beta_t E_t \left[c_{t+\tau} - \frac{Q_{t+\tau}\, x_{t+\tau}^{1-\iota}}{\iota - 1} \right] + (1-\alpha)\delta^\iota\, \beta_t E_t c_{t+\tau}; \quad \text{and} \tag{A.14}$$

$$S_t - \frac{Q_t y_t^{1-\iota}}{\iota - 1} = \alpha \beta_t E_t \left[S_{t+\tau} - \frac{Q_{t+\tau}\, y_{t+\tau}^{1-\iota}}{\iota - 1} \right] + (1-\alpha)\delta^\iota\, \beta_t E_t c_{t+\tau}; \tag{A.15}$$

which yields a pair of equations for c_t and S_t in terms of the thresholds x_t and y_t and the exogenous variable Q_t. The solution for x_t, y_t, c_t, and S_t is determined by (A.5), (A.6), (A.14), and (A.15), with the exogenous variable Q_t obtained from (A.2).

Given y_t, the value of p_t comes from equation (8), and x_t and T_t from (19). The laws of motion involve equations (20) and (21) for S_t and u_t. Considering equation (22) for new listings N_t, make the following definitions of a new variable ξ_t and a constant γ:

$$\xi_t = (1-\gamma)\sum_{\ell=0}^{\infty} \gamma^\ell\, u_{t-\tau}; \quad \text{where } \gamma = \alpha + (1-\alpha)\delta^\iota; \tag{A.16}$$

Using this new variable, equation (22) for listings becomes

$$N_t = (1-\alpha)(1 - u_{t-\tau} + S_{t-\tau}) - \frac{m(1-\alpha)\delta^\iota}{(1-\gamma)}\, x_t^{-\iota}\, \xi_{t-\tau}; \tag{A.17}$$

Equation (A.16) defining ξ_t can be stated equivalently as follows:

$$\xi_t = \gamma \xi_{t-\tau} + (1-\gamma)u_t;$$

where a variable without a time subscript denotes the steady-state value of that variable. Equation (A.5) implies the steady-state moving threshold x and surplus S are related as follows:

$$x + F = \frac{m}{t} S;$$ (A.21)

The steady-state thresholds y and x are linked in accordance with equation (A.6):

$$y = ab\, x + \frac{1 - ab}{t}\, C;$$ (A.22)

The steady-state value of c can be deduced from equation (A.14):

$$c = \frac{x^{1-l}}{(l - 1)} \frac{t}{1 - yb};$$ (A.23)

where $y = a + (1 - a)d^l$ is as defined in (A.16). A relationship between S and c can be derived using

Combined with $N = m \bar{y}^{1-\mu} u$, this can solved for the steady state u

$$u = \frac{(1-a)}{(1-a) + m\,a\,y^{1-\mu} + d^l x^{1-\eta}\frac{(1-a)}{(1-y)}} = \frac{1}{1 + m\frac{a}{1-a}y^{1-\mu} + \frac{d^l}{1-y}x^{1-\eta}}; \tag{A.26}$$

The steady state implied by the price equation (A.19) is:

$$P = kC\left[b\frac{\tau}{1-b}D + w\frac{1-b(1-mp)}{1-b}\frac{\tau}{m}\frac{x+F}{p}\right]; \tag{A.27}$$

which uses (A.21) to substitute for S.

Log linearizations Log deviations of variables from their deterministic steady-state values are denoted using sans serif letters, for example $x_t = \log x_t - \log x$. The log linearization of equation (A.2) for Q_t is

$$\mathsf{q}_t = (1-ab)\,\pi_t + ab\,\mathsf{b}_t + ab\,E_t\,\mathsf{q}_{t+t};$$

which uses the steady-state values $q = 1$ and Q from (A.20). The discount factor is $b_t = e^{-\tau r_t}$ in terms of the discount rate r_t, and $b = e^{-\tau r}$ is its steady-state value. It follows that $\mathsf{b}_t = \log b_t - \log b = -\tau(r_t - r) = -\tau r_t$, where $\mathsf{r}_t = r_t - r$ is the deviation of the discount rate from its steady-state level. The log-linearized equation for Q_t can then be written as

$$\mathsf{q}_t = (1-ab)\,\pi_t - ab\tau\,\mathsf{r}_t + ab\,E_t\,\mathsf{q}_{t+t}; \tag{A.28}$$

Noting (A.20) and (A.21), the log linearization of the moving-threshold equation (A.5) is

$$\mathsf{x}_t = ab\,E_t\mathsf{x}_{t+t} + (1-ab)\frac{(x+F)}{x}\,\pi_t - (1-ab)\,\pi_t; \tag{A.29}$$

The transaction threshold (A.6) can be log linearized as follows:

$$\mathsf{y}_t = \frac{x}{y}ab\,(E_t\,\pi_{t+t} + E_t\mathsf{x}_{t+t} - \tau\mathsf{r}_t) - \pi_t; \tag{A.30}$$

and this equation can be used to deduce that

$$\mathsf{y}_t - ab\,E_t\mathsf{y}_{t+t} = \frac{x}{y}ab\,(E_t[\pi_{t+t} - ab\,E_{t+t}\pi_{t+2t}] + E_t[\mathsf{x}_{t+t} - ab\,E_{t+t}\mathsf{x}_{t+2t}] - \tau(\mathsf{r}_t - ab\,E_t\mathsf{r}_{t+t}))$$

$$(\pi_t - ab\,E_t\,\pi_{t+t}) = \frac{x}{y}ab\,E_t\left((1-ab)\,\pi_{t+t} - ab\tau\,\mathsf{r}_{t+t}) + (1-ab)\frac{(x+F)}{x}\pi_{t+t} - \pi_{t+t}\right)$$

$$+ \frac{x}{y}ab\tau(\mathsf{r}_t - ab\,E_t\mathsf{r}_{t+t}) - ((1-ab)\,\pi_t - ab\tau\,\mathsf{r}_t)$$

$$= \frac{(x+F)}{y}(1-ab)ab\,E_t\,\pi_{t+t} - (1-ab)\,\pi_t + \frac{(y-x)}{y}ab\tau\,\mathsf{r}_t; \tag{A.31}$$

where the subsequent expressions follow from substituting (A.28) and (A.29).

For equation (A.14) for c_t, by using (A.20) and (A.23), the log linearization is

$$\mathsf{c}_t = a + (1-a)d^l - b\,E_t\,\pi_{t+t} + \frac{1-yb}{1-ab}\left((\pi_t - ab\,E_t\,\pi_{t+t}) + (1-l)(\mathsf{x}_t - ab\,E_t\mathsf{x}_{t+t})\right)$$

$$a + (1-a)d^l - a - \frac{1-yb}{1-ab}b\tau\,\mathsf{r}_t;$$

and with the de nition of $y = a + (1-a)d^I$

Using (A.21) and (A.27), the price equation (A.19) is log linearized as follows:

k

with exogenous moving predicts that the volatility of new listings is tiny relative to sales, while empirically, new listings is more volatile than sales. Listings also have a perfectly negative correlation with houses for sale in the model, but are almost uncorrelated in the data. As a result, listings and houses for sale have same correlations (in absolute value) with other variables. The correlation with sales is also much lower than found in the data. The problem is simply that listings are proportional to the previous number of homeowners not trying to sell because a fraction of these homeowners receive an idiosyncratic shock that leads them automatically to try to sell irrespective of market conditions. Thus listings can only vary as a re ection of changes in house for sale, and by a much smaller amount.

Figure 8: Impulse responses to a housing demand shock with exogenous moving
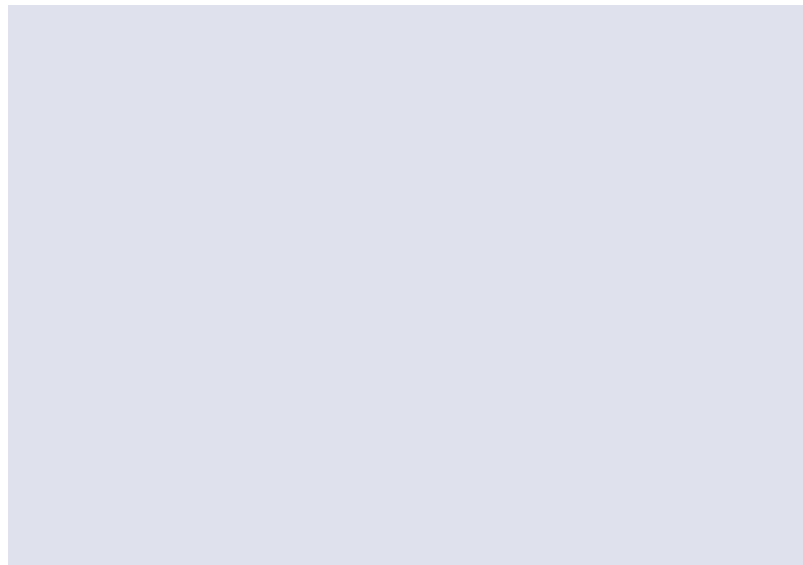
Notes: The model with exogenous moving is the special case $\lambda = 0$. The housing-demand shock has persistence given by $f_q = 0.9873^{1=13}$.

The problems faced by the model with exogenous moving extend beyond simply the behaviour of new listings. Compared to the data, the relative volatility of sales is far too low. The reason for this failing can be seen in the impulse response functions in Figure 8. While the shock to the demand for housing pushes up sales, with no possibility of signi cant in ows, these sales quickly deplete the stock of houses for sale, persistently reducing the stock of properties on the market. This then offsets the effect of the demand shock on sales because fewer sales take place when few properties are available, even if the selling rate remains high (and so time-to-sell remains persistently shorter). Because there is no margin for more than the usual number of homeowners to enter the market as sellers, the shift in demand leads to excessive volatility in the number of houses for sale and time-to-sell. The predicted correlations between sales and new listings, and sales and prices are both too low compared to the data.