



# Mini-Conference on Infinite Combinatorics 2012

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LSE Mathematics Department 25<sup>th</sup> July 2012

Adam Ostaszewski, LSE

Title: Steinhaus' Theorem and its descendants

Abstract: The classical real-line Steinhaus sum-theorem that  $A+A$  contains an open set when  $A$  is large in the sense of measure or category, has generated a corpus of work which will be recalled and continues to inspire, among them results on automatic continuity in group theory and open mapping principles in analysis. Such results may be deduced from the notion of shift-compactness associated with group action. In the presence of almost completeness this property follows from the separation of points from closed nowhere dense sets by appropriate group members. Variant forms of separation may be deduced by passing to a refinement topology; thus the density topology permits measure results to follow from category results. Analyticity provides a useful mild form of completeness. Mention will be made of the relationship between this approach and the use of generic automorphisms in the sense of Truss.

Dona Strauss, Hull

Title: Chains of idempotents in  $\beta \mathbb{N}$  (Joint work with N. Hindman and Y. Zelenyuk)

Abstract: The properties of idempotents in  $\beta \mathbb{N}$  play a significant role in combinatorics. They have often provided surprisingly short proofs of important theorems - Hindman's Theorem and the Hales-Jewett Theorem are examples - and they have suggested new theorems. I shall describe what we know about idempotents in  $\beta \mathbb{N}$  and talk about a new result obtained by N. Hindman, Y. Zelenyuk and myself about the existence of decreasing chains of idempotents in  $\beta \mathbb{N}$ . We showed that, for every non-minimal idempotent  $p$  in  $\beta \mathbb{N}$  and every countable ordinal  $\alpha$ , there is a decreasing chain of idempotents in  $\beta \mathbb{N}$  indexed by  $\alpha$ , with  $p$  as the maximum element. Whether there are any uncountable chains of idempotents in  $\beta \mathbb{N}$  is an open question.

An audio recording (with accompanying screenshots) of this presentation is available at [http://richmedia.lse.ac.uk/math/20121107\\_SlawomirSolecki.mp4](http://richmedia.lse.ac.uk/math/20121107_SlawomirSolecki.mp4).

**Slawomir Solecki, University of Illinois at Urbana-Champaign**

Title: An abstract approach to Ramsey theory with applications

Abstract: The classical Ramsey theorem states that for any finite coloring of the  $k$ -element subsets of  $\mathbb{N}$ , there is an infinite monochromatic subset. This theorem has been generalized in many ways, and one of the most interesting generalizations is the abstract version of Ramsey's theorem. This version states that for any finite coloring of the  $k$ -element subsets of a set  $X$ , there is an infinite monochromatic subset of  $X$  which is closed under the operation of taking  $k$ -element subsets. This theorem has been generalized in many ways, and one of the most interesting generalizations is the abstract version of Ramsey's theorem. This version states that for any finite coloring of the  $k$ -element subsets of a set  $X$ , there is an infinite monochromatic subset of  $X$  which is closed under the operation of taking  $k$ -element subsets.